

Comparing Extreme Wave Estimates from Hourly and Annual Data

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ABSTRACT

This paper estimates 100-year wave height levels from (1) a model fit to all wave heights observed over 18 years in a Northern North Sea location; and (2) an extreme event model that considers only the 18 annual maxima. The result of method (1) is generally found to exceed that from method (2). We seek here to reconcile this difference, considering the effects of clustering, statistical and model uncertainty. The general conclusion is to favor approaches that directly model the large wave height events of interest; e.g., annual maxima or storms. Beyond its relevance to extreme waves, this study aims to show useful results to quantify statistical uncertainty and clustering effects in estimating extremes.

INTRODUCTION

A basic problem in reliability analysis is to estimate extreme load fractiles from limited data. In general there is a tradeoff between (1) global models based on all data; and (2) extreme event models, based on a subset of the largest loads available (e.g., annual maxima). While the global approach (1) utilizes all available data, it can obscure critical tail behavior and introduce correlation among observations (e.g., clustering). In contrast, extreme events in approach (2) may be more nearly independent, but their scarceness increases statistical uncertainty.

This study shows convenient analytical methods to quantify these effects of clustering and statistical uncertainty. They are applied here to a measured North Sea wave data set, in which 100-year wave height estimates from approaches (1) and (2) are found to differ. By reconciling the difference in this case, we seek to supplement various studies of extreme wave heights, and their uncertainties (e.g., Guedes Soares and Moan, 1983; Olufsen and Bea, 1990; Winterstein and Haver, 1991). More broadly, we hope to shed light on the general effects of dependence and uncertainty on extreme value estimation.

GLOBAL AND EVENT MODELS OF EXTREME WAVES

The wave elevation $\eta(t)$ at a fixed location is typically assumed stationary over a fixed “seastate” interval T (here, $T=3$ hours). Over this period, the intensity of $\eta(t)$ is commonly reported through the significant wave height, $H_s=4\sigma_\eta$, in terms of the standard deviation, σ_η , of $\eta(t)$. Further, for ocean engineering analysis and design against extreme waves, it has become common to seek the 100-year level h_{100} , the H_s level which is exceeded with a mean return period of 100 years.

While observed H_s histories may span multiple years, significant extrapolation generally remains to estimate h_{100} . Either an extreme event or a global wave model may be used:

1. We may consider extreme H_s values in fixed periods; e.g., the annual maximum height H_{ann} in various years. Using observed maxima to fit the the distribution function $F_{ann}(h)=P[H_{ann} \leq h]$, h_{100} is defined as

$$1 - F_{ann}(h_{100}) = .01 \text{ per year} \quad (1)$$

2. We may instead use all H_s data to fit $F_{3-hr}(h)$, the distribution of H_s when sampled in an arbitrary 3-hour seastate. In this case h_{100} is found not from Eq. 1 but from

$$\begin{aligned} 1 - F_{3-hr}(h_{100}) &= \frac{.01}{N} = \frac{.01}{2920} \\ &= 3.42 \times 10^{-6} \text{ per seastate} \end{aligned} \quad (2)$$

Here N reflects the number of 3-hour seastates per year: $N=365 \times 8=2920$.

Note that these two approaches should yield similar h_{100} results if various 3-hour H_s values are identically distributed and statistically independent. With these assumptions,

$$F_{ann}(h) = P[H_1 \leq h \text{ and } \dots \text{ and } H_N \leq h] = [F_{3-hr}(h)]^N \quad (3)$$

If we choose h_{100} to satisfy Eq. 2, Eq. 3 implies that $F_{ann}(h_{100})=(1 - .01/N)^N$, or about $e^{-.01} \approx 0.99$ in agreement with Eq. 1.

In practice, however, these two methods do not always yield similar results. We illustrate this with measured wave data from a Northern North Sea location, which span roughly 18 years. (In a closing section, we will revisit these results based on an extended field data set

that spans roughly 26 years, consisting of measured data from Statfjord, Troll, Brent, Stevenson, and Gullfaks C. In all cases, note that all reported wave height data have been *measured* in the North Sea; no hindcast wave heights have been used here.)

Choosing first to model all 3-hour seastates, we fit a conventional three-parameter Weibull model of $F_{3-hr}(h)$:

$$F_{3-hr}(h) = 1 - \exp\left\{-\left(\frac{h-h_0}{h_c}\right)^\gamma\right\}; \quad h \geq h_0 \quad (4)$$

The parameter values $h_0=0.59$, $h_c=2.27$, and $\gamma=1.40$ have been used here to preserve the first three moments of the data.

Figure 1 compares the resulting estimate of F_{3-hr} with the data, plotted on “Weibull scale” ($-\ln[1 - F_{3-hr}]$ vs. h on log-log scale). A two-parameter Weibull model ($h_0=0$) will plot as a straight line on this scale. The resulting h_{100} estimate is 14.5m, found from Eq. 2 by setting $-\ln(1 - F_{3-hr})=12.58$. Note also that the fit appears reasonably good. Figure 2 reveals the discrepancy between global and extreme wave event models. It shows the 18 annual maximum H_s values, which range from $h_1=8.7$ m to $h_{18}=12.1$ m, with the associated distribution estimates $F_{ann}(h_i)=i/19$ ($i=1\dots 18$). In contrast, if we use the Weibull fit of $F_{3-hr}(h)$ from Eq. 4, the resulting annual distribution $F_{ann}(h)=F_{3-hr}(h)^N$ seems to overestimate various fractiles of annual maxima.

To estimate h_{100} directly from the observed annual maxima, we fit these data to a Gumbel model:

$$F_{ann}(h) = \exp[-e^{-\alpha(h-u)}] \quad (5)$$

Rewriting this in terms of the mean and standard deviation, $\mu=u + .577/\alpha$ and $\sigma=1.28/\alpha$, of the Gumbel variable, an arbitrary fractile is given by

$$h_p = \mu + \sigma K_p \quad (6)$$

in terms of the standardized Gumbel variable

$$K_p = -.45 - .78 \ln(-\ln p) \quad (7)$$

Thus, the Gumbel model in Figure 2 yields a linear variation between h and $[-\ln(-\ln F_{ann})]$. The 100-year wave height is then estimated from Eq. 6 by setting $p=.99$ ($K_p=3.14$), and replacing μ and σ by the sample moments of the observed maxima. As shown in Figure 2, the resulting estimate is $h_{100}=13.2$ m. Note that this value is roughly 10% below the estimate, $h_{100}=14.5$, found from all seastate data.

Finally, note that when fitting probability distributions to data, we consistently apply the method of moments. This method is used here to seek a reasonable fit within the body of the data. It is not our intent to systematically compare results across various fitting methods; e.g., maximum likelihood and least squares methods. We have, however, some experience applying these other methods to the wave data sets at hand. While precise numerical values may vary, the various methods generally show the same trends; i.e., reduced h_{100} estimates when only annual data are considered.

EFFECT 1: CLUSTERING

A natural first hypothesis is that the estimate $h_{100}=14.5$, found from all seastate data, is too high due to the independence assumption in Eq. 3. Dependence between H_s values in successive 3-hour seastates will generally reduce expected extremes: if one H_s value is less than the level of interest, neighboring values will be more likely to be less as well due to dependence. However, this effect is not expected to be large at the high

H_s levels of interest, for which clusters of multiple crossings should be quite rare.

To quantify this effect, we assume that the probability that the next wave height, H_{i+1} , is less than h_{100} depends on whether the last wave height, H_i , was less—and not *additionally* on still earlier wave heights H_{i-1}, H_{i-2}, \dots . This is a form of Markovian, one-step memory in time. Under this assumption, Eq. 3 is amended to read

$$\begin{aligned} F_{ann}(h) &= P[H_1 \leq h] \cdot P[H_2 \leq h | H_1 \leq h] \cdots P[H_N \leq h | H_{N-1} \leq h] \\ &= P[H_1 \leq h] \cdot \{P[H_{i+1} \leq h | H_i \leq h]\}^{N-1} \end{aligned} \quad (8)$$

Here we propose the following form of the conditional probability distribution (Appendix A):

$$P[H_{i+1} \leq h | H_i \leq h] = 1 - A(x)e^{-x}; \quad A(x) = \frac{1 - e^{-cx}}{1 - e^{-x}} \quad (9)$$

In this result, the constant c depends on the correlation ρ between H_i and H_{i+1} , while x is defined as

$$x = \left(\frac{h-h_0}{h_c}\right)^\gamma \quad (10)$$

In the independent case $\rho=0$ and $c=1$, and Eqs. 8–10 reduce to Eqs. 3–4. In the other limiting case of perfect dependence, $\rho=1$ and $c=0$, so that the conditional probability in Eq. 9 becomes 1 as well.

Figure 3 compares the results of the independent and clustered models (Eqs. 3 and 8). Eq. 9 is used with $c=0.13$, which is found from Eq. 19 to preserve the observed correlation $\rho=.95$ between neighboring H_s values. As expected, this dependence has little effect on extreme wave height estimates: h_{100} reduces from 14.5m to about 14.2m. (Similarly small effects are found for extreme responses of dynamic systems, where clustering effects have been widely studied and quantified.) Various alternative clustering models, fit to data in each season or with a different conditional distribution than Eq. 9, give h_{100} estimates from 13.9–14.3m. Thus, clustering effects do not appear sufficient to explain the entire difference of 14.5–13.2 or 1.3m between our original h_{100} estimates.

Finally, before proceeding we may note that the foregoing results can be used to derive a general expression for N_{indep} , an “equivalent number of independent observations” that can be used with a simple independent wave height model. If independence is assumed, the annual maximum distribution is as given in Eq. 3:

$$F_{ann}(h) = \{P[H_i \leq h]\}^{N_{indep}} \quad (11)$$

Equating Eqs. 8 and 11 and solving for N_{indep} , we find

$$N_{indep} = 1 + (N - 1) \frac{\ln[1 - A(x)e^{-x}]}{\ln[1 - e^{-x}]} \quad (12)$$

The limiting cases are consistent: when $\rho=0$, $c=A(x)=1$ and $N_{indep}=N$, the actual number of data. Alternatively, when $\rho=1$, $c=A(x)=0$ and Eq. 12 yields $N_{indep}=1$ (i.e., all subsequent data are perfectly correlated). More generally, note that N_{indep} is a function of the level x of interest; as the level $x \rightarrow \infty$, $N_{indep} \rightarrow N$ (as should be the case). Figure 4 shows the general trend of N_{indep} with wave height level; for example, at $h=14.5$ m N_{indep} is roughly 80% of the total number of data. Note also that for high thresholds ($e^{-x} \ll 1$), Eq. 12 is well-approximated (Figure 4) by

$$\frac{N_{indep} - 1}{N - 1} \approx A(x) \approx 1 - e^{-cx} \quad (13)$$

where, again, c depends on the correlation coefficient ρ . For the typical results considered here, the expression $c \approx 1 - \rho^{2.7}$ may serve as a simple

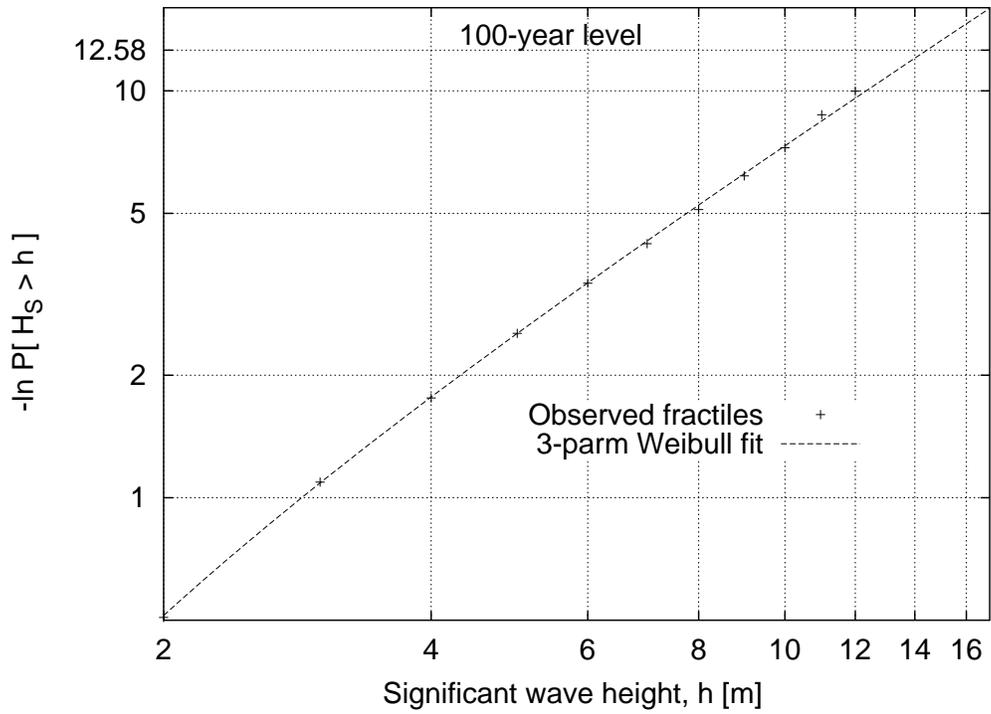


Figure 1: Observed seastate distribution, Weibull scale.

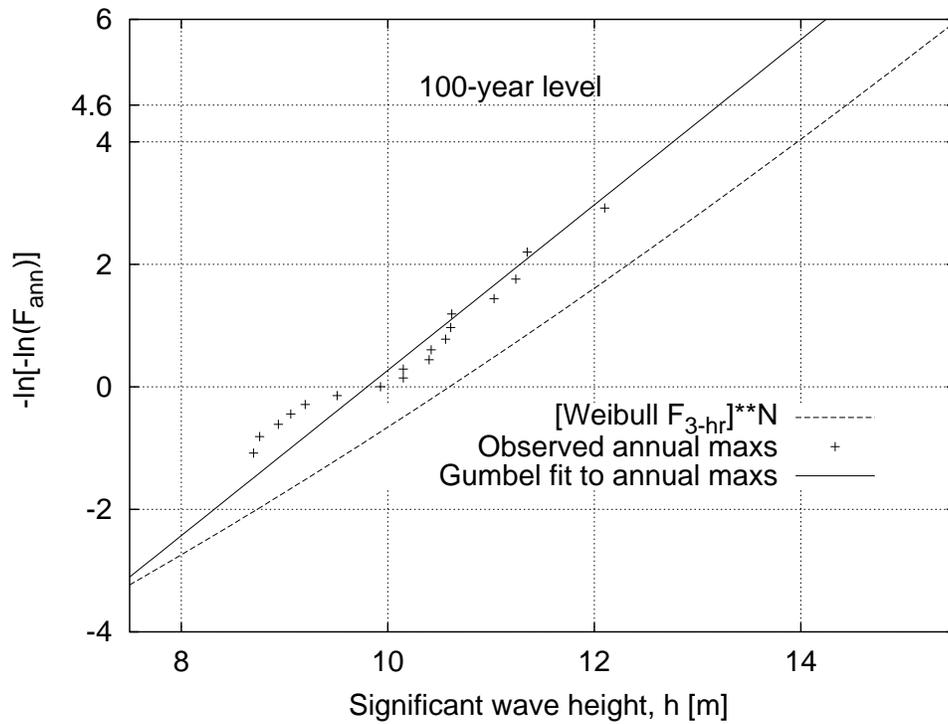


Figure 2: Annual maximum distribution, Gumbel scale.

first approximation. If interest lies in the wave height with exceedance probability p_e per seastate, setting $p_e \approx e^{-x}$ leads to the result

$$\frac{N_{indep} - 1}{N - 1} \approx A(x) \approx 1 - p_e^c \quad (14)$$

For example, for the 100-year seastate, $p_e = 3.42 \times 10^{-6}$ (Eq. 2) and $N_{indep}/N \approx .80$ as noted above.

EFFECT 2: STATISTICAL UNCERTAINTY

A second hypothesis instead favors the estimate $h_{100}=14.5\text{m}$, because it is based on all seastate data. In contrast, one may question the estimate $h_{100}=13.2\text{m}$ because it is based on a limited sample of 18 annual maxima. Specifically, how likely is it that another statistically equivalent set of 18 maxima could produce an estimate of 14.5m or above?

This can be estimated analytically, if we assume the Gumbel model in Eq. 6 has correct form but is based on imperfect moment estimates, $\hat{\mu}$ and $\hat{\sigma}$, from n data. The uncertainty in these moments can be estimated (Appendix B) as follows:

$$\text{Var}[\hat{\mu}] = \frac{\sigma^2}{n}; \quad \text{Var}[\hat{\sigma}] \approx \left(\frac{\alpha_4 - 1}{4}\right) \frac{\sigma^2}{n}; \quad \text{Cov}[\hat{\mu}, \hat{\sigma}] \approx \frac{\alpha_3}{2} \frac{\sigma^2}{n} \quad (15)$$

Using these first two moments to predict an arbitrary fractile h_p (Eq. 6), the resulting uncertainty is

$$\text{Var}[h_p] = \frac{\sigma^2}{n} \left[1 + K_p^2 \left(\frac{\alpha_4 - 1}{4} \right) + K_p \alpha_3 \right] \quad (16)$$

in which $K_p = (h_p - \mu)/\sigma$. Note that Eqs. 14 and 15 are completely general, and can be applied to any distribution (normal, Weibull, Gumbel, etc.). The first term in Eq. 15 reflects the effect of uncertainty in the mean, the second the uncertainty in the standard deviation, and the last the correlation between their estimates.

For our h_{100} estimate, we apply Eq. 15 with the sample variance $\hat{\sigma}^2 = .94$, the higher moments $\alpha_3 = 1.14$ and $\alpha_4 = 5.4$ of the Gumbel model, and $K_p = 3.14$ (Eq. 7 with $p = .99$). The result is

$$\text{Var}[h_p] = \frac{.94}{18} [1 + 3.14^2 (1.10) + 3.14 (1.14)] = .81 = [0.9\text{m}]^2 \quad (17)$$

Note that the dominant term here is due to uncertainty in σ , which dominates uncertainty in the mean by about an order of magnitude.

Thus, the estimate $h_{100}=14.5\text{m}$ from all seastates is about 1.4 standard deviations above the level $h_{100}=13.2\text{m}$ estimated from annual maxima. This difference appears rather significant: a difference as large as this would appear only 8% of the time if we assume the moments in Eq. 14 to have joint normal distribution.

EFFECT 3: MODEL UNCERTAINTY

Note that we have not considered statistical uncertainty in the alternate estimate $h_{100}=14.5\text{m}$. This is because it is based on roughly 40,000 seastate data, which should permit accurate estimation of the Weibull distribution parameters in Eq. 4. One may question, however, the adequacy of the Weibull model itself, particularly in the upper tail region of interest here. Figure 5 addresses this point. It repeats both analytical models of F_{ann} from Figure 2, which lead to $h_{100}=13.2$ and 14.5m . The higher value arises from $F_{ann} = F_{3-hr}^N$, with the Weibull model of F_{3-hr} from Eq. 4. Also shown is F_{3-hr}^N in terms of the *observed* F_{3-hr} values for all data

above 8m. Note that the Weibull model agrees well with the data below 10m; however, the data suggest a narrower tail above that level. (This trend can also be seen in Figure 1, although obscured by both the scale and the use of 1-meter data bins.)

Extrapolation of these data would appear to favor $h_{100}=13.2$ over 14.5m . As a separate confirmation, we also consider a ‘‘storm’’ approach, which models the peak H_s value following every excursion of the threshold $h_{th}=8\text{m}$. We find a mean number of $N_{storm}=78/18=4.3$ storms per year. Figure 5 shows the annual maximum distribution implied by these storm data: $F_{ann}=F_{storm}^{N_{storm}}$, with the observed storm distribution $F_{storm}(h_i)=i/79$ at each of the storm heights h_i .

The storm results strongly support the Gumbel model of annual maxima, and again suggest that the Weibull model overestimates the upper tail. In fact, a Gumbel model of all storms gives virtually the same h_{100} estimate as the annual maxima alone:

$$h_{100} = \mu + 4.29\sigma = 13.2\text{m}; \quad \sigma_{h_{100}} = 0.55\text{m} \quad (18)$$

This h_{100} estimate uses Eq. 6 with the observed storm mean and standard deviation, and $K_p=4.29$ from Eq. 7 with $1 - p = .01/4.3$ (mean rate of storms per 100 years). The uncertainty in h_{100} is estimated from Eq. 15. It is somewhat smaller than that based on annual maxima (Eq. 16), because the storm approach utilizes more data than the annual maxima alone. (Note that we have also considered various thresholds other than 8m to define storms. In general, as the storm threshold increases from small levels (e.g., 5–6 m), corresponding h_{100} estimates decrease rather steadily, leading to results that approach h_{100} estimates based on the annual maxima only.)

EXTENDED DATA SET RESULTS

The foregoing are the results of a study completed several years ago, while the second author was a visiting scholar at Stanford University. More recently, we have had the chance to revisit these analysis results, based on an extended North Sea data set that covers roughly 26 years (over the 1973–1999 period). Figure 6 is an updated version of Figure 2. Now the difference between h_{100} estimates has closed: $h_{100}=14.9\text{m}$ (fitting all data) vs $h_{100}=14.1\text{m}$ (fitting annual maxima). The difference has nearly halved: $14.9-14.1=0.8\text{m}$ compared with $14.5-13.2=1.3\text{m}$ previously.

Because the net difference is less, clustering effects can serve to explain a greater fraction: using the same correlated exponential model, we again estimate a clustering reduction of 0.3m ($14.9-14.6$, as opposed to $14.5-14.2$ originally). In fact, Figure 7 shows a variety of clustering results. Beyond the correlated exponential model, results of a NATAF model (e.g., Der Kiureghian and Liu, 1986) are also displayed. Also shown is the ‘‘observed conditional $[F_{3hr}(h)]^N$,’’ where F_{3hr} is the direct data estimate of the conditional probability $P[H_{i+1} < h | H_i < h]$ (as sought by the Markov model; see Eq. 9). These conditional data tend to support the adequacy of the correlated exponential fit, at least in this case.

CONCLUSIONS

- Several methodological developments have been shown to aid in estimating extreme values and their uncertainty. For example, a Markovian model of successive wave heights has been suggested to quantify the effect of dependence on extremes (Eqs. 8–10). This can be applied to either all data or each season separately. Also, general

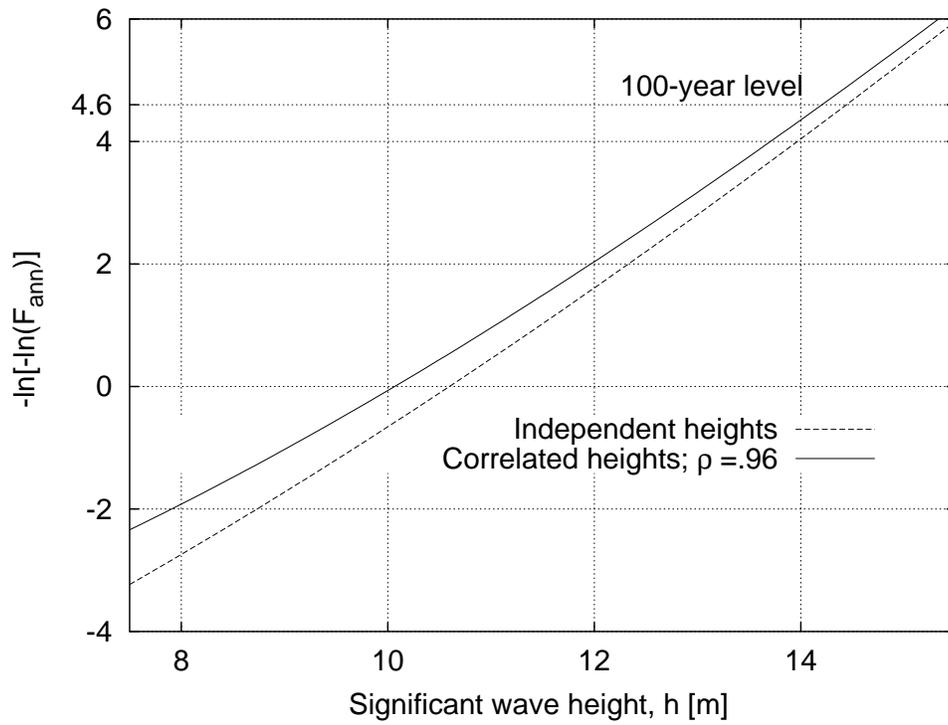


Figure 3: Effect of H_s dependence.

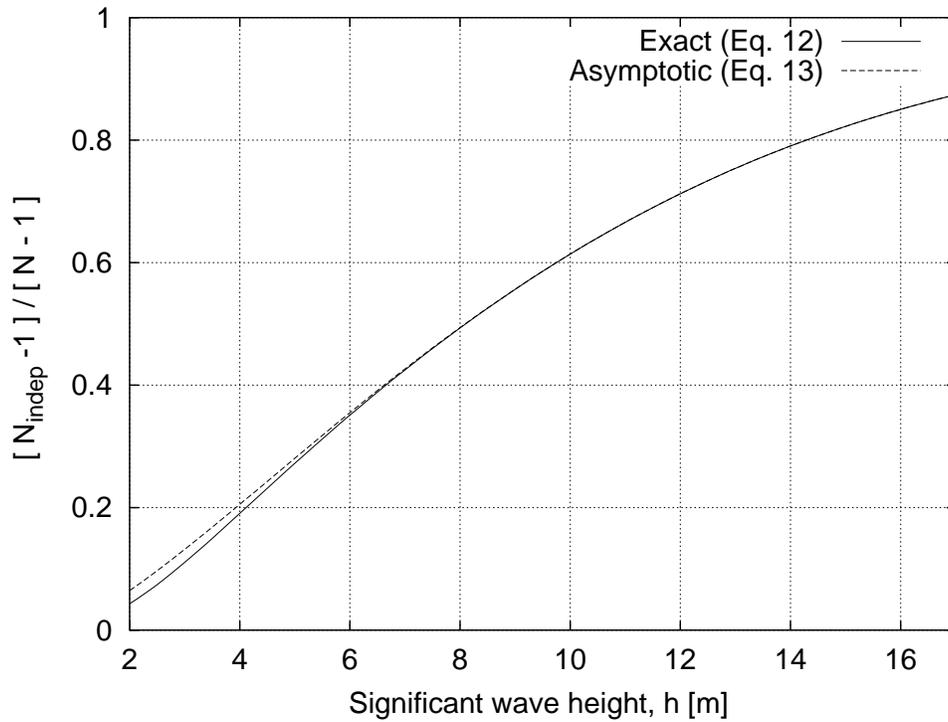


Figure 4: Equivalent number of independent wave height data (from Eq. 12 or Eq. 13) as a function of the wave height level of interest.

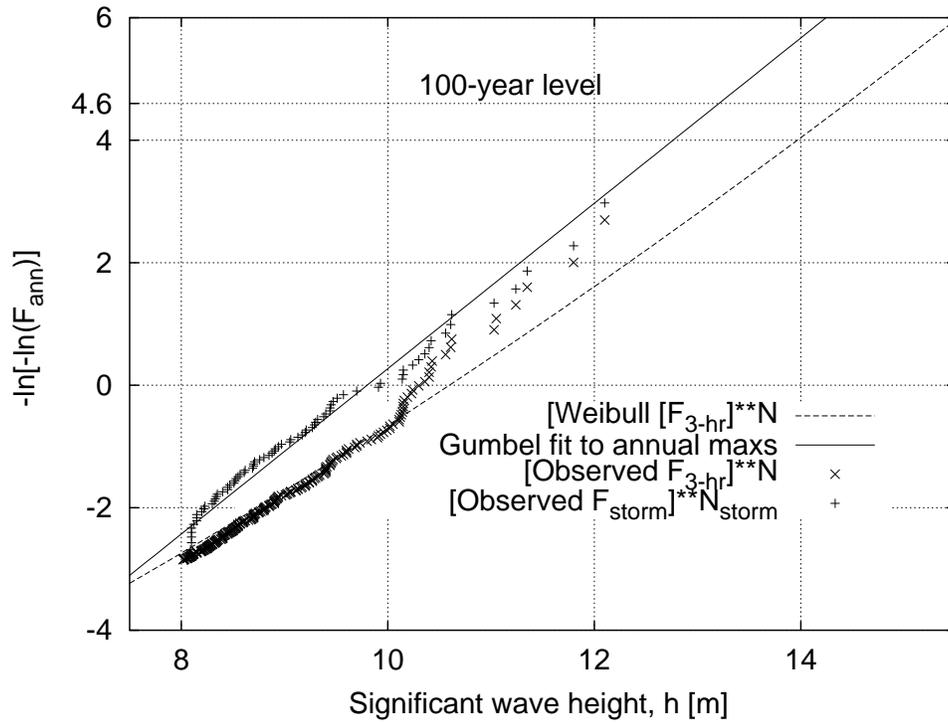


Figure 5: Upper tail of seastate data; storm model.

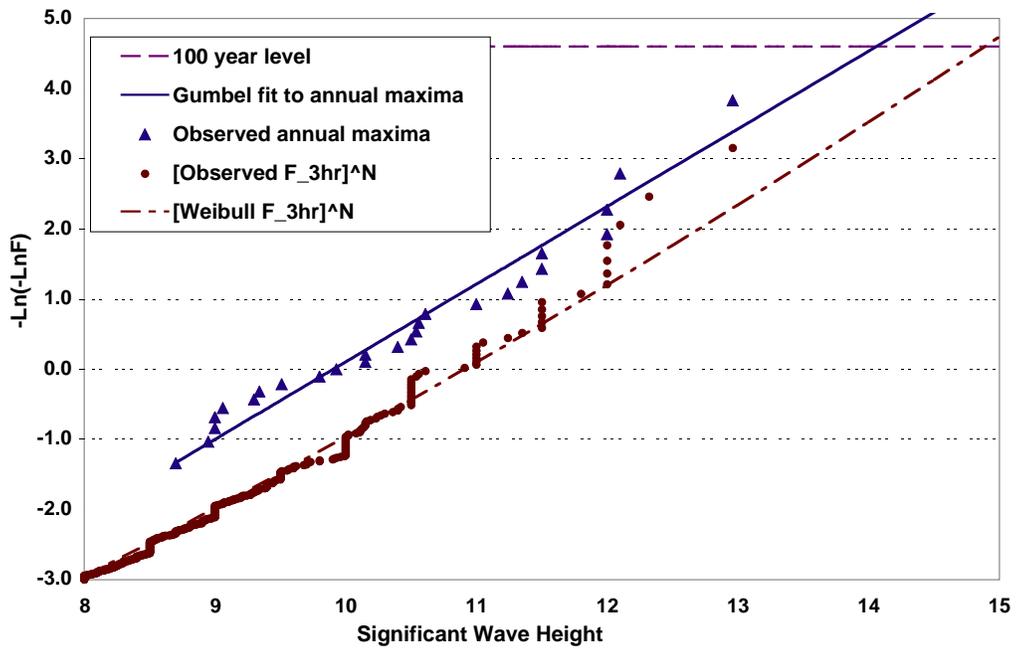


Figure 6: Predicted annual maximum distribution, based on (1) Weibull fit to all data and (2) Gumbel fit to annual maxima. (Results based on extended, 26-year data set.)

results have been shown to quantify statistical uncertainty in arbitrary response fractiles, when predicted by the method of moments (Eq. 15).

- We have applied these methods here to study the difference between 100-year wave height estimates based respectively on annual maxima and all seastate data. For the 18-year data set, it did not appear that this difference $\Delta h=14.5-13.2=1.3\text{m}$ was easily explained by wave height dependence (Figure 3), or by statistical uncertainty due to the limited sample of 18 annual maxima (Eq. 16). For the extended 26-year data set, the difference reduces to $\Delta h=14.9-14.1=0.8\text{m}$ (Figure 6), or $\Delta h=14.6-14.1=0.5\text{m}$ after clustering is included (Figure 7). This difference may more plausibly lie within the range of statistical uncertainty.
- The general conclusion is to favor approaches that directly model the large wave height events of interest; e.g., annual maxima or storms. While one gains more data by including all seastates, a global fit to all such data may appear quite accurate (Figure 1) yet obscure critical upper tail behavior (Figure 5). (This is one reason we have retained the 18-year case; it provides an equally plausible data set, for which these effects are more dramatic.) Note that in these applications the global Weibull seastate model appears to introduce conservative errors; however, nonconservative errors could arise in other applications.

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APPENDIX A: BIVARIATE WEIBULL DISTRIBUTION

If two correlated variables X_i and X_{i+1} both have standard exponential marginal distributions, a convenient joint distribution function is (Johnson and Kotz, 1969):

$$F_{X_i, X_{i+1}}(x, y) = 1 - e^{-x} - e^{-y} + \exp[-(x^m + y^m)^{1/m}] \quad (19)$$

Here the parameter m determines the correlation ρ_x between these variables. Independence corresponds to $m=1$, while perfect correlation ($\rho_x=1$) is achieved as $m \rightarrow \infty$. If we set $y=x$ and divide by $F_{X_i}(x)=1-e^{-x}$, the conditional distribution in Eq. 9 is found (with $c=2^{1/m}-1$).

If Eq. 10 defines x values in terms of actual wave heights h , x will have standard exponential distribution if the wave height distribution follows Eq. 4. Finally, given an assumed value of m , the correlation between H_i and H_{i+1} can be found by solving

$$E[H_i H_{i+1}] = \int \int h(x_i) h(x_{i+1}) f(x_i, x_{i+1}) dx_i dx_{i+1} \quad (20)$$

in which $h(x)$ is found by inverting Eq. 10, and $f(x_i, x_{i+1})$ by differentiating Eq. 18. Trial and error then gives the value of m , and hence $c=2^{1/m}-1$, that produces the desired correlation between H_i and H_{i+1} .

APPENDIX B: JOINT MOMENT UNCERTAINTY

Given observations $X_1 \dots X_n$, it is common to estimate the mean by $\hat{\mu}=\sum X_i/n$, and the variance by $V=\sum(X_i-\hat{\mu})^2/(n-1)$. The exact moments of $\hat{\mu}$ and V are (Fisher, 1928):

$$\text{Var}[\hat{\mu}] = \frac{\sigma^2}{n} \quad (21)$$

$$\text{Cov}[\hat{\mu}, V] = \alpha_3 \frac{\sigma^3}{n} \quad (22)$$

$$\text{Var}[V] = \frac{\sigma^4}{n} \left(\alpha_4 - 1 + \frac{2}{n-1} \right) \approx (\alpha_4 - 1) \frac{\sigma^4}{n} \quad (23)$$

Because fractile results such as Eq. 6 use the sample standard deviation, $\hat{\sigma}=V^{1/2}$, rather than V directly, a Taylor series for $\hat{\sigma}=V^{1/2}$ is useful:

$$\hat{\sigma} = V^{1/2} \approx \bar{V}^{1/2} + \frac{1}{2} \bar{V}^{-1/2} (V - \bar{V}) \quad (24)$$

in which $\bar{V}=E[V]=\sigma^2$. Combining this result with Eqs. 20–22, the results in Eq. 14 are found.

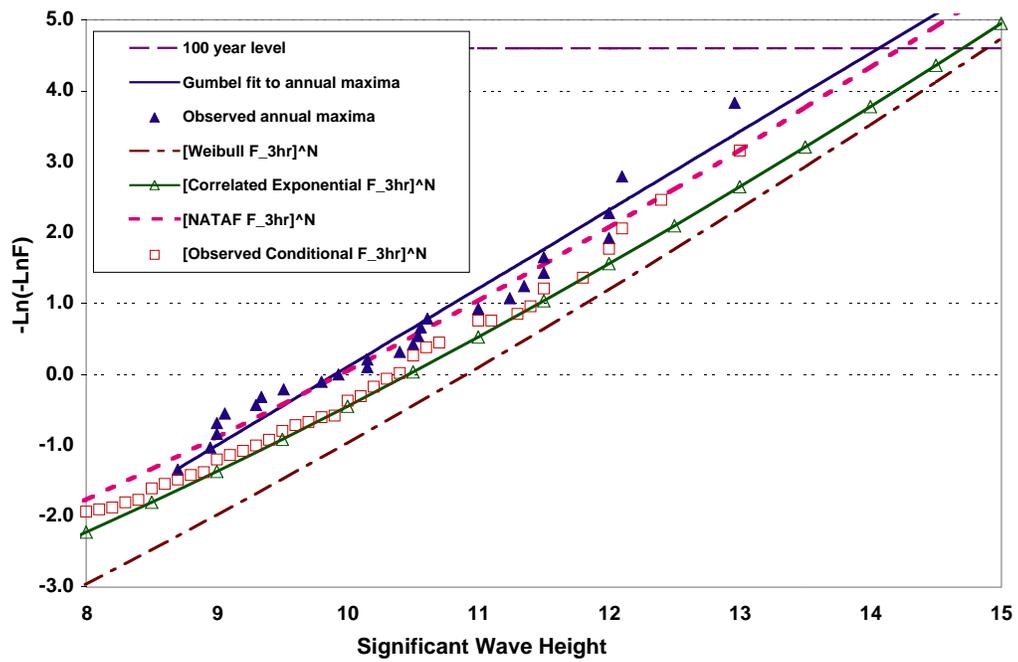


Figure 7: Various corrections for clustering effects. (Results based on extended, 26-year data set.)