

The Modal Distribution Method: A New Statistical Algorithm for Analyzing Measured Acceleration Data

Bert Sweetman and Myoungkeun Choi

Texas A&M University at Galveston, 200 Seawolf Parkway, Galveston, Texas, USA

ABSTRACT

A new statistical method is proposed to quantify the significance of changes in mean frequencies of individual modal vibrations of measured structural response data. In this new method, called the modal distribution method, a power spectrum of measured structural response resulting from a Fourier transform is interpreted as being a series of independent modal responses. Each modal response is isolated over a frequency range and treated as a statistical distribution. The first two spectral moments are calculated directly from each of these distributions. A combined statistical comparison of the means of modal frequencies in separate data windows is used to produce a quantitative significance level of the observed differences between power spectra. Significant changes between these spectra indicate a change in the underlying process, such as damage detection in a structural health monitoring application. The method is general and may find a broad variety of applications, but it seems particularly well suited for structural health monitoring applications because the excitation is not required as input.

An example is presented based on measured full-scale acceleration data from a drilling riser. To validate the new method, a power spectrum resulting from the field data is idealized to a target spectrum with known mean and variance of each mode. The idealized spectrum is subtly changed and new acceleration time-histories are simulated from these modified spectra to assess the effectiveness of the new method. The modal distribution method is found to be very effective at detecting subtle changes of mean modal frequencies.

Keywords: Statistical approach, Acceleration data, Modal vibrations, Modal analysis, Structural health monitoring, Random vibrations, Spectral moments, Statistical moments, Statistical significance, Modal distribution method

1. INTRODUCTION

Analysis of measured sensor data is an increasingly important topic because of the decreasing cost and size as well as the increasing capability of sensor and computer hardware. In civil, mechanical, and aerospace engineering, analysis of measured vibration response is increasingly used for monitoring or assessing structural condition. Structural monitoring using integrated sensors often leads to vast amounts of data and associated heavy computational and communication loads. Stochastic analysis can transform large amounts of data into compact values representing system vibration characteristics, which can be used for detecting changes in structural condition.

Experimental modal analysis is the process of using experimental data to determine the modal parameters of a structure for each mode of vibration. In conventional experimental modal analysis, the modal parameters are extracted from a set of frequency response function measurements. Conventionally, determining these frequency response measurements requires that both the excitation force and vibration response must be known. ^{e.g. 1,2} In many practical applications, the excitation force is unknown, which makes use of these traditional methods impossible. In such cases, white-noise excitation is sometimes assumed, ^{e.g. 3,4} but in some applications such an assumption may be unreasonable. Here, a new method is proposed that requires only structural response measurements.

The proposed method compares modal response characteristics computed for separate segments of a measured time-history. The new method has two important advantages over more conventional analyses: 1) Only a

Bert Sweetman: E-mail: sweetman@tam.u.edu, Telephone: (409) 740-4834
Myoungkeun Choi: E-mail: mkchoi@tam.u.edu, Telephone: (409) 740-4505

measured time-history of the response is required as input, and 2) the result is a numerical prediction of the confidence level that there has been some change in the modal response of the structure. This significance level indicates the confidence that structural response has in fact changed, which is a proxy for whether or not the structure itself has changed.

2. OVERVIEW OF THE NEW METHODOLOGY

A measured acceleration time-history is divided into a series of segments, or windows, which are to be sequentially analyzed. Each segment of the time history is converted into a power spectrum using a Fourier transform. It is implicitly assumed that the dynamic behavior of a structure in a given frequency range can be considered as a set of individual modes of vibration and that these vibrational modes can be considered to be independent. Hinging on that assumption, the power spectrum is divided into individual modes, as represented by “humps” in the power spectrum, using a penalty approach. Each isolated individual mode is then treated as a distribution of energy around a mean modal frequency, with the variance representing the spread of the energy around that frequency. Rescaling each mode into a statistical distribution enables use of conventional statistical tools for comparison between distributions resulting from separate windows in the time-history. Changes observed in individual modal distributions between segments in the time-history are believed to be indicative of changes in either structural stiffness or mass. Finally, the significance of these changes is detected by statistical comparison.

This overview first describes the new methodology from a conceptual standpoint with only minimal computational detail. Computational detail is presented in Section 3.

2.1. Separation of Modal Distributions

In general, a discrete power spectrum is composed of a series of small discrete frequency intervals, each with an associated power found between the upper- and lower-frequency bounds. Such a spectrum is typically represented as a histogram in which each bar represents the energy between two frequencies, as shown in Figure 1.

The behavior of individual modal frequencies is of primary interest here. Accordingly, modal distributions are isolated and then treated individually as probability density functions. The methodology by which the modes are isolated requires an initial guess for each of the modal frequencies. The frequency which optimally separates the two modes, i.e. the frequency of the bottom of the trough, or N_i in Figure 1, is determined through application of a penalty method. After these local minima have been determined, all energy between two adjacent minima is assumed be associated with a single mode of vibration. Using this method, a mean frequency and associated region of the power spectrum is identified for each mode of vibration. The shape of each resulting region represents the distribution of energy around the mean frequency.

2.2. Modal Distributions and Probability Density Functions

Next, the region surrounding each of the M modal frequencies is isolated and considered independently. The probability of energy within any one frequency range can be computed as the area of one frequency bar divided by the total energy associated with that i 'th mode:

$$P_i(e_n) = \frac{a_n}{A_i} \quad (1)$$

where a_n is the area of n 'th frequency bar, and A_i is the total energy between the upper and lower local minima surrounding the i 'th modal frequency. This representation implicitly considers each modal frequency as a random variable and each frequency bar surrounding the modal frequency as part of a discrete probability density function (PDF).

Mapping of a power spectrum into probability space is precisely the inverse of simulating a time series using a power spectrum with random phase. A time-history is commonly simulated from a target power spectrum as:

$$x(t) = \sum_{n=1}^N H_n \cos(2\pi f(n)t + \theta_n) \quad (2)$$

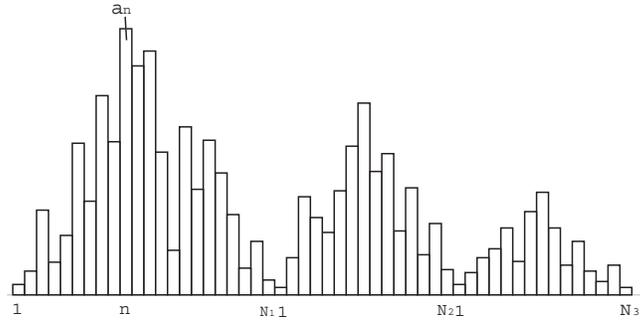


Figure 1. Example Discrete Spectrum

where $x(t)$ is the time-history of the simulation, H_n is the amplitude of each frequency component, which is related to the energy at that frequency, $f(n)$ is the n 'th circular frequency (Hz), and θ_n is a random phase angle.

Any frequency component with zero energy ($a_n = 0$) does not exist in the resulting simulated time-history, which means there is zero probability of energy in that frequency range. Conversely, if an infinitely long sinusoidal time series is converted to a power spectrum, it is represented as having only one frequency bar, which includes the sinusoidal frequency.

After each region has been rescaled into probability space, the result is a series of individual modal frequency distributions. The mean and variance of each distribution can be calculated directly from the geometry of the distribution. The mean is a representative modal frequency, and the variance indicates the spread of energy around that modal frequency. The total area of the distribution prior to normalization represents the energy associated with that vibrational mode. The resulting variance of a particular mode is not the variance of the process, but is instead only the part of the variance associated with structural vibration at that particular mode.

2.3. Comparison: Statistical Hypothesis Testing

The overall goal is to detect subtle changes in the state of a vibrating structure by comparing modal frequency distributions between different segments of a measured response time-history. The comparison of two distributions is performed by statistical comparison of means and variances of the distributions. The entire procedure to calculate the mean and variance of each mode could be repeated over consecutive segments (windows) of a time-history or could be applied real-time to monitor for structural failure. The mean of each sequential modal distribution in the first window are statistically compared with those of the second window, i.e, the mean of the first mode of the first window are compared with that of the first mode of the second window, as are each of the subsequent modal means. The statistical significance of any changes in the mean and variance is reported. The magnitude of the energy associated with each vibrational mode is used only to weight the mean and variance when computing an overall T -statistic to predict the confidence that the vibrational characteristics of the structure have in fact changed.

3. COMPUTATIONAL DETAILS

3.1. Separation of Modal Distributions

As outlined in Section 2.1, the behavior of individual modal frequencies is of primary concern. Power spectra of real measured data are not smooth, and identifying the correct minima to use as dividing points between modal distributions is non-trivial. The methodology presented here relies on an initial estimation of the frequency of each mode of vibration for the structure. This initial value would be expected to come from structural analysis, though it could also come from visual inspection of the power spectrum. This initial value need not be of high accuracy because it is only used to specify the endpoints of the penalty method. The method searches for one local minimum between every two adjacent modal frequencies, plus one additional minimum above the highest and one below the lowest specified frequencies.

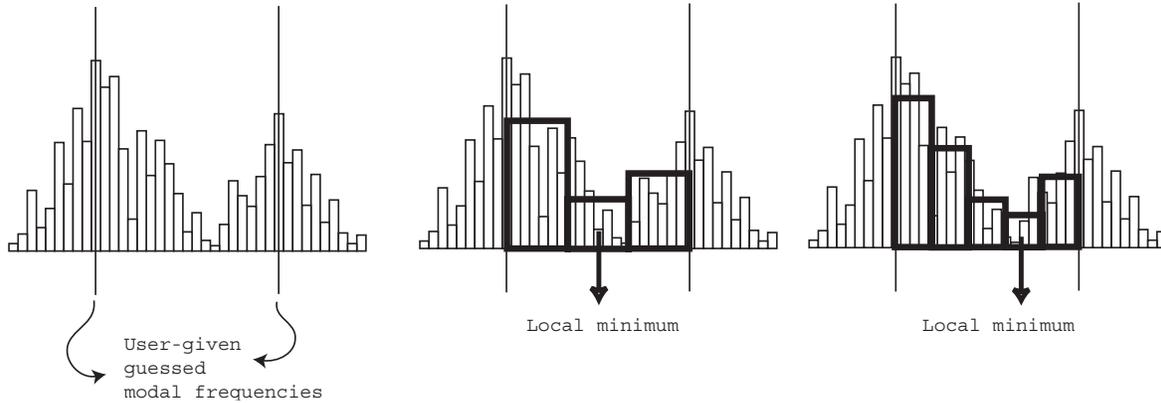


Figure 2. Separation of Modal Distributions

In searching for each local minimum, the penalty method searches the region between two adjacent modal frequency guesses by progressively seeking the minimum of increasingly fine division of the region. First, the region is divided into three equal frequency intervals as shown in the second frame of Figure 2. The total area under the power spectrum within each of the three intervals is then computed; every frequency in that interval having the lowest average energy is assigned a penalty value of 1.0. Next, the number of divisions between initial modal frequency guesses is increased to four. The same penalty approach is applied: again, every frequency in the region having the smallest average energy (lowest average bar height) receives a penalty; on this second iteration the penalty is 0.9.

The process is repeated through a total of ten iterations, with the number of divisions increasing by 1 and the penalty decreasing by 0.1 with each subsequent penalty assessment. Beyond four intervals, a penalty is applied to each frequency in every interval having lower average energy (lower average bar height) than both of its contiguous neighbors. The total penalty is then calculated for each frequency between the initial modal frequency guesses. The frequency having the largest total penalty is the local minimum between modes and is used as the dividing point between modal frequencies. All energy between adjacent local minima is assumed to be associated with a single vibrational mode. Once these modes have been isolated, distribution parameters are calculated for each mode.

3.2. Modal Distributions and Probability Density Functions

As outlined in Section 2.2, modal frequencies are treated as random variables. The energy surrounding each modal frequency is treated as a probability distribution of candidates for the true modal frequency of the underlying structural vibration.

Equation 1 outlined the normalization by which the modal distributions are mapped into probability distributions:

$$P_i(e_n) = \frac{a_n}{A_i}$$

where a_n is the area of n 'th frequency bar, and A_i is the total energy between the upper and lower local minima surrounding the i 'th modal frequency. The total energy associated with the mode is given by:

$$A_i = \sum_{n=1}^{N_i} a_n = \sum_{n=1}^{N_i} S(n)df \quad (3)$$

where $S(n)$ is the magnitude of the power spectrum at the n 'th frequency and df is the frequency spacing.

The first two spectral moments of each resulting distribution are treated as the first two moments of a PDF: the mean and variance. These moments are computed directly from the offsets of the power spectrum using conventional definitions of moments and central moments. e.g. 5,6

$$\mu_{i,w} = \frac{m_i}{A_i} = \frac{1}{A_i} \sum_{n=1}^{N_i} S(n) f(n) df \quad (4)$$

$$s_{i,w}^2 = \frac{\theta_i}{A_i} = \frac{1}{A_i} \sum_{n=1}^{N_i} S(n) (f(n) - \mu_{i,w})^2 df \quad (5)$$

where $\mu_{i,w}$ and $s_{i,w}^2$ are the mean frequency and sample variance of the modal distribution, respectively, m_i is the first moment, θ_i is the second central moment, $S(n)$ is the spectrum magnitude at the n 'th frequency, $f(n)$ is the n 'th frequency, df is the frequency interval, and N_i is the total number of bars within the i 'th modal distribution.

Some local variation in computed values of the mean and variance of each mode is expected. There are two important sources of variations in the mean and variance, the first of which is that the data-set being sampled is finite: If one were to calculate e.g. the mean and variance in the weight of apples in a truck, the mean and variance might change considerably as the sample size increases from five apples to six, but would generally not be affected very much by an increase from 1,000 apples to 1,001. This source of variation is accounted for in the T -static applied in Section 3.3. In this application to random vibrations, however, there is a second potentially important source of variation: sampling of incomplete modal cycles, which is not considered in the T -statistic.

Consider computing the variance of a portion of a single sine wave as in Equation 5. If there happened to be exactly one complete cycle, the power spectrum would appear as a spike and variance would be zero; if, however, there happened to be 1.5 cycles of the sine wave, the power spectrum would not appear as a spike and the computed variance would be non-zero. For certain applications with a limited number of modal cycles (i.e. a short time-history relative to the longest modal period), this effect could be very important. This effect was investigated for the example presented in Section 4, however, and found to be relatively small. A typical difference between the mean variance averaged over 100 data-points, each of which was computed for successive points in the time-history, and the minimum over the same 100 points was calculated. The difference is about 0.6% for the first mode in a one-hour time-history and about 0.3% in a two-hour time-history. In the example presented in Section 4, the A_i , $\mu_{i,w}$ and $s_{i,w}^2$ calculated from Equations 3–5 are averaged over the last 100 points in the time-history. The resulting small over-prediction of the modal variance is believed to be negligible.

3.3. Comparison: Statistical Hypothesis Testing

Actual field data does not generally exhibit a clear spectral peak at each modal frequency. Instead, spectral peaks appear as a cluster of energy around some frequency. Here, mean modal frequencies are treated as random variables and observed clusters of energy are treated as statistical distributions. The benefit is that rigorous statistical analysis can then be applied to assess the significance of apparent differences between distributions. The significance of the difference depends on the mean, variance and number of degrees of freedom of the two distributions being compared.

The T -test is a conventional method to determine the statistical significance of a difference between two sample means. The test can be used when the number of samples is too small for the Central Limit Theorem to apply and where the true variances of the underlying processes are not known to be equal. The T statistic is:

$$T_i = \frac{\Delta\mu_i}{s_i} \quad (6)$$

where

$$\Delta\mu_i = \mu_{i,1} - \mu_{i,2} \quad (7)$$

$$s_i^2 = s_{i,1}^2/N_{i,1} + s_{i,2}^2/N_{i,2} \quad (8)$$

$$N_{i,w} = T_w \mu_{i,w} \quad (9)$$

in which $\mu_{i,w}$ is the mean of the i 'th mode of the w 'th segment, $s_{i,w}^2$ is the sample variance, and the number of samples $N_{i,w}$ is estimated as the number of cycles expected at each modal frequency. T_w is the duration of the w 'th window.

The statistic computed in Equation 6 is distributed approximately as a Student's T with the number of degrees of freedom for the i 'th mode equal to:

$$DOF_i = \frac{(s_{i,1}^2/N_{i,1} + s_{i,2}^2/N_{i,2})^2}{(s_{i,1}^2/N_{i,1})^2/(N_{i,1} - 1) + (s_{i,2}^2/N_{i,2})^2/(N_{i,2} - 1)} \quad (10)$$

where $s_{i,w}$ is the sample standard deviation of the i 'th mode for the w 'th window, and $N_{i,w}$ is the number of cycles in the i 'th mode.

3.4. Overall Comparison of Response

Assessing the significance of changes in the overall observed vibrational response requires simultaneous comparison of all modes. This comparison of the two distributions treats the differences between modal frequency means as repeated measurements with different uncertainties. An overall P-value is calculated by computing a combined T-statistic by weighting differences between the means by the fraction of energy represented by each individual mode. The fraction of energy associated with the i 'th mode for each of the two w 'th window is:

$$E_{i,w} = \frac{A_{i,w}}{\sum_{i=1}^I A_{i,w}} \quad (11)$$

where I is the total number of modes in the power spectrum. The values of the mean and variance of the i modes are combined as weighted averages to compute the T -statistic:

$$T = \frac{\Delta\mu}{s} \quad (12)$$

where

$$\Delta\mu = \sum_{i=1}^I 0.5(E_{i,1} + E_{i,2})\Delta\mu_i \quad (13)$$

$$s^2 = \sum_{i=1}^I [0.5(E_{i,1} + E_{i,2})s_i]^2 \quad (14)$$

The resulting single T-statistic represents the weighted average differences between the two distributions, individually considering all of the modes. To compare this T -statistic with a standard T -distribution, the total number of degrees of freedom must be calculated. Physically, this total represents the sum total of the number of vibration cycles in each individual mode. Generally, this number of degrees of freedom will be larger than the observed number of peaks in the time-history because modal frequencies with small fractions of the total energy will not contribute to the number of peaks in the time-history. The total degrees of freedom is estimated as the sum of individual degrees of freedom, corrected for prior removal of the degree of freedom associated with each T -statistic in Equation 10:

$$DOF = \sum_{i=1}^I DOF_i + I - 1 \quad (15)$$

The resulting degrees of freedom are used in conjunction with the T -static to compare with a standard Student's T -distribution to predict the statistical significance level. This P-value represents the probability that the observed differences between the two power spectra are due to random chance rather than actual differences between the underlying processes.

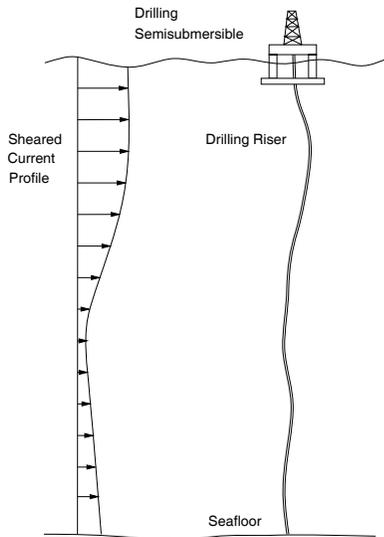


Figure 3. Drilling Semisubmersible

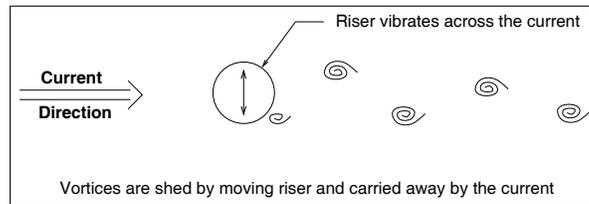


Figure 4. Formation of Vortices.

4. EXAMPLE: MARINE DRILLING RISER VIBRATIONS

Prediction of fluid-structure interaction relevant to marine risers has been a historically intractable problem. In very deep waters, oil is produced to floating platforms where one of the biggest engineering challenges is design of marine risers, which are the pipes transmitting fluids between the sea-floor and the floating platform. Marine risers present major design challenges because only the riser top and bottom are externally restrained (Figure 3). The unsupported span equals the water depth and can easily be deformed by ocean currents.

A riser, or any other slender structure subject to strong flow across its axis, may interact with the current to create vortex-induced vibrations (VIV). Even slight structural motions across the current can cause a vortex to be shed on one side of the pipe. As the vortex is carried away by the current, the pipe is pulled in the direction of the departing vortex; the resulting motion then creates a vortex on the opposite side (Figure 4). A strong oscillatory process develops if the vortex shedding frequency is close to a natural frequency of the pipe. A detailed investigation of the VIV motions of risers on a North-Sea platform leading to gas leakage concluded that despite extensive work, a predictable model for description of the VIV phenomena has not yet been developed.⁷

In high current deep water areas, stress resulting from vortex-induced vibrations is a primary driver for fatigue damage.⁸ Unfortunately, quantifying riser response has proven difficult. A detailed comparison of four numerical models to predict VIV motions found major discrepancies between results and concluded that not all aspects of VIV are well understood.⁹ The energy associated with a vibrating marine riser is relatively free to travel along the riser and sudden shifts in modal vibration frequencies are sometimes observed in measured data. It may be that a key step to understanding the physical process associated with riser VIV is identifying specific times in measured field data at which the riser response shifts between modal vibration patterns.

One possible application of the proposed new method is identifying these vibrational shifts. In this example, the effectiveness of the new method is demonstrated using simulated data based on measured field data. Simulated data was chosen over measured field data because the mean and variance of the underlying process are known and can be controlled, so the effectiveness of the method can be meaningfully assessed.

4.1. Simulated Data

The data used in this example was simulated as follows: First, the power spectrum of measured field data was computed. The first two modes of this measured spectrum were then smoothed into the ideal target spectrum S shown in Fig. 5. A subtle change was then applied to the target spectrum, which would be representative of a change in the structural behavior. The change that was introduced was a small shift in the mean and/or variance of each of the two modal response distributions. In every case, shifts affecting the mean of one mode

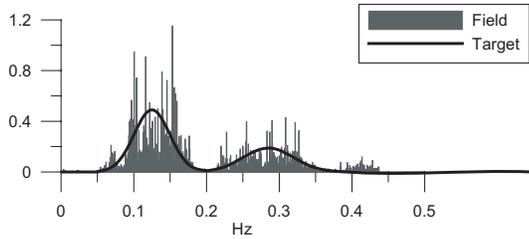


Figure 5. Field Data Spectrum and Idealized Target Spectrum S

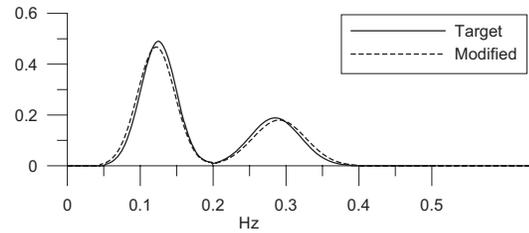


Figure 6. Original Spectrum (S) and Modified Spectrum (S_{m2v1})

were offset by shifts in the opposite direction affecting the other mode such that all target power spectra have the same mean. For example, in case S_{m1} the mean of the first mode was shifted downward by 2% and the second mode was shifted upward by 1.6% so the combined response retains the same mean. Changing modal distributions of target spectra while retaining the overall mean precludes effective detection by conventional tests of stationarity such as the runs and reverse arrangements tests. In case of variance, a 10% increase in both modes was introduced, while holding the total energy of the spectra constant. Time-histories were then simulated from these target spectra (e.g. Figures 7 and 8). Each resulting simulated time-history has mean and variance comparable to the original measured time-history in both the overall spectrum and in each vibrational mode, but these means and variances of the underlying processes are precisely known for the simulated data. Target spectra means and variances for each mode of all spectra are shown in Table 1. In the following example, the new method is applied to each of these test cases for increasingly long data windows; within each case, the same irregular time-history is used. For example, the 1 hour data window test for target spectrum S_{m1} includes the segment of the time-history used in the 30 minute test.

Table 1. Target Spectra Parameters with Percent Change from Original Idealized Spectrum

Target Spectrum	1st Mode				2nd Mode			
	Mean	% Change	Variance	% Change	Mean	% Change	Variance	% Change
S	0.12500	N/A	0.00060	N/A	0.28500	N/A	0.00120	N/A
S_{m1}	0.12250	-2%	0.00060	-	0.28950	+1.6%	0.00120	-
S_{m2}	0.12188	-2.5%	0.00060	-	0.29069	+2%	0.00120	-
S_{m3}	0.12125	-3%	0.00060	-	0.29182	+2.4%	0.00120	-
S_{v1}	0.12500	-	0.00066	10%	0.28500	-	0.00132	10%
S_{m1v1}	0.12250	-2%	0.00066	10%	0.28950	+1.6%	0.00132	10%
S_{m2v1}	0.12188	-2.5%	0.00066	10%	0.29069	+2%	0.00132	10%
S_{m3v1}	0.12125	-3%	0.00066	10%	0.29182	+2.4%	0.00132	10%

4.2. Mean Shift

This first example of the new method examines the capability to detect subtle shifts in the mean of the underlying process if the variance of each mode is held constant (target spectra S_{m1} through S_{m3}). A 5% P-value indicates that there is only a 5% probability that the observed differences between the distributions result from random chance alone; differences with P-values less than 5% are commonly regarded as statistically significant. Figures 9 and 10 show the P-values resulting from separate comparisons of the first and second modes. For the first mode, a 30 minute window is inadequate to detect a shift of the mean as small as 3%, but shifts as small as 2 or 2.5% can be detected if a data-window of 2 hours is available.

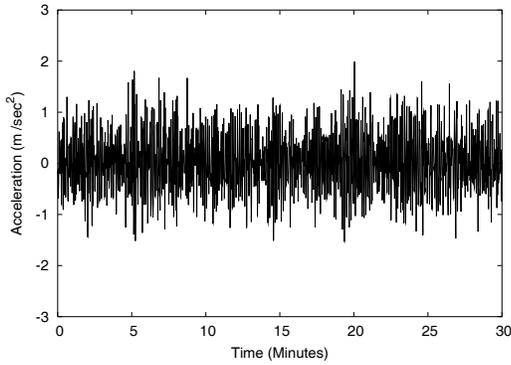


Figure 7. Sample time-history simulated using the baseline idealized target spectrum (S)

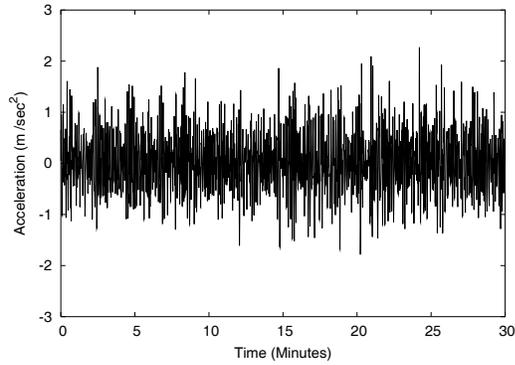


Figure 8. Sample time-history simulated using the modified target spectrum (S_{m2v1})

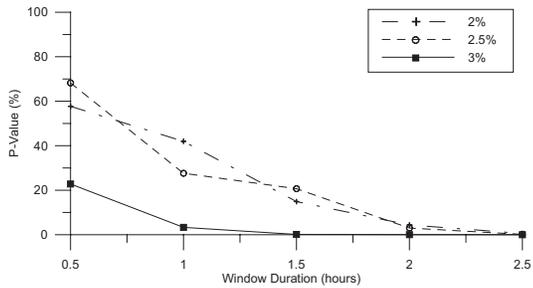


Figure 9. Mean Change Only (1st Mode)

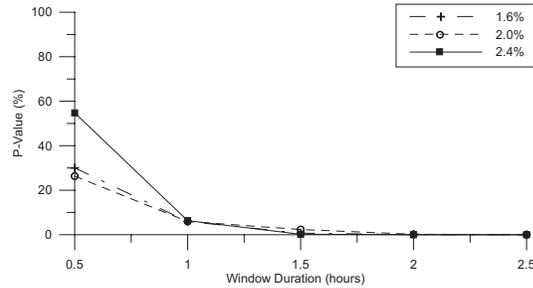


Figure 10. Mean Change Only (2nd Mode)

Figure 11 is included as a caution. It shows additional results for the same case as Figures 9 and 10. Here, the P-value is calculated for data windows increasing by 2.5 minute increments. It is readily apparent that significance levels do not decrease monotonically when the P-values are very high. Generally, high P-values are less stable than low P-values. Fortunately, only low P-values, those less than about 5%, are regarded as statistically significant and as such only those cases are of more than academic interest.

4.3. Variance Increase

In the second test, the mean frequency of each mode and the total energy of the process are held constant, while each variance is increased by 10% (target spectrum S_{v1}). As expected, the T -test shows no statistically significant difference between means, but there is a noticeable decreasing trend in the P-values of the second mode. This trend is believed to be an indirect effect of the assumption that *all* energy between the local minima surrounding a mode is associated with that mode (and that is the only energy associated with that mode). If each mode in fact has the general shape of a T-distribution, then the energy under the lower tail of the distribution associated with the upper mode is counted with the lower mode rather than the upper. Cutting off the lower

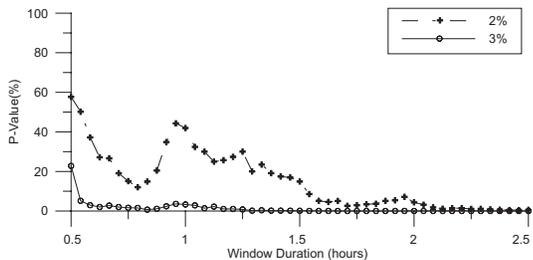


Figure 11. Scatter in High P-Values

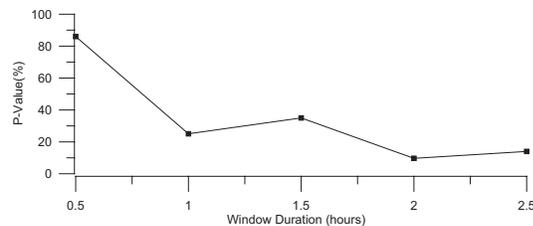


Figure 12. Variance Change Only (2nd Mode)

tail of the upper distribution effectively causes its observed mean to shift slightly upward, resulting in gradually declining confidence that the true underlying mean has not changed.

4.4. Combined Change

This final part of the example is the most realistic set of simulated data in that both the mean and variance are changed, as would be typical in measured data (target spectra S_{m1v1} through S_{m3v1}). As can be seen by comparing Figures 9 and 10 with Figures 13 and 14, the increase in variance does not have any substantial effect on the method's ability to detect modal frequency shifts. Finally, Figures 15 and 16 demonstrate the overall capability to detect changes in the power spectra combining all data available from both modes using Equations 12–15. Figure 16 repeats the data from Figure 15, but the vertical scale is adjusted to show only those values within the 5% significance level. Generally, use of all available predictors (the observed shifts in both modes) would be expected to yield the lowest overall P-value. However, in the 30 minute data window the second moment alone has a more compelling P-value than the combined because of the very high P-value for the first modal comparison.

The P-values shown in these figures correspond to the changes to the idealized target spectra shown in Figure 6, which are subtle even in idealized form. The actual time-series are shown in Figure 7 and 8; these two time-histories are nearly indistinguishable by eye, as they would be by conventional statistical methods which do not individual modal behavior. They are nearly indistinguishable because the underlying processes for the full power spectra have *identical* mean and variance. However, using the new modal distribution method, only a 30 minute data-window is required to differentiate these two spectra with only a 3.6% chance that the apparent differences are due to random chance alone, and a 0.25% chance given a one hour data window. For even longer data windows, the computed P-values suggest virtual certainty that these time-series result from different underlying processes.

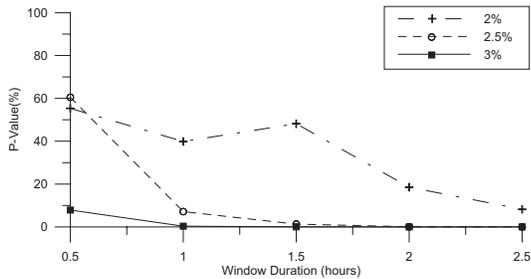


Figure 13. Mean and Variance Change (1st Mode)

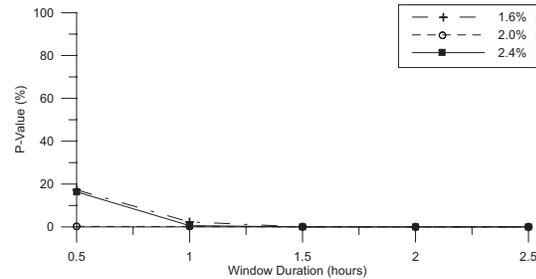


Figure 14. Mean and Variance Change (2nd Mode)

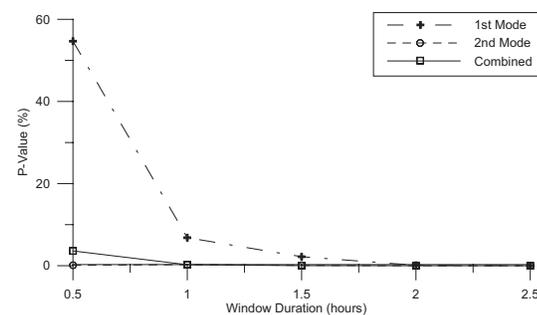


Figure 15. Overall P-values for both modes: Combined Mean and Variance Change

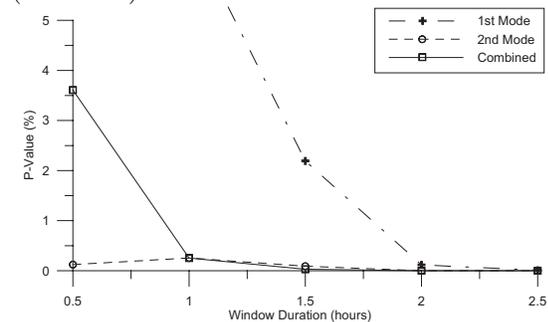


Figure 16. Overall P-values for both modes: Combined Mean and Variance Change

5. CONCLUSIONS

A newly proposed modal distribution method of quantifying the significance of subtle changes in modal vibrations based on power spectra has been presented and shown to be very effective at detecting changes in mean frequencies of individual modal vibrations in simulated data (Figure 16). In the modal distribution method, a power spectrum of measured structural response resulting from a Fourier transform is interpreted as being a series of individual independent modal responses. A penalty approach is employed to define the ambiguous separation points between modal energy in the power spectrum (Figure 2). Each modal response is isolated over a frequency range in the power spectrum and treated independently as a statistical distribution (Equation 1). The spectral moments are directly calculated from the geometry of each of the resulting distributions (Equations 4–5). A statistical comparison of the difference in means of each modal frequency pair observed in two time segments results in a quantitative significance level of the observed differences between the power spectra based on isolated modal distributions (Equations 12–15). The modal distribution method is general and is applicable to any number of modes of vibration and so may find a broad variety of applications. It seems particularly well suited for structural health monitoring applications because the excitation is not required as input.

An example is presented, which is based on measured full-scale acceleration data from a marine drilling riser. To validate the new method, a power spectrum resulting from field data is idealized to a target spectrum having only two modes of vibration (Figure 5), each with known mean and variance. The idealized spectrum is subtly changed (Figure 6) and new acceleration time-histories are simulated from the modified ideal spectra to assess the effectiveness of the new method. The new modal distribution method is found to be very effective at detecting subtle changes of mean modal frequencies (Figure 16). Only a 30 minute data window is required to detect the subtle change to the target spectrum shown in Figure 6 with a 3.6% confidence level, and given data windows greater than one hour, the difference in target spectra is detected with virtual certainty.

6. NOMENCLATURE

$\Delta\mu$ Weighted average difference between mean frequencies over all modes in all data windows

$\Delta\mu_i$ Difference between mean frequencies of the i 'th mode

μ_i Mean frequency of the i 'th mode

$\mu_{i,w}$ Mean frequency of the i 'th mode in the w 'th window

θ_i The second central moment of the i 'th modal distribution

A_i Total area of the i 'th mode in a window

$A_{i,w}$ Total area under the power spectrum associated with the i 'th mode in the w 'th window

a_n Area under the power spectrum associated with the n 'th frequency in a single mode

DOF Degrees of Freedom for T -statistic

DOF_i Degrees of Freedom for the T -statistic for i 'th mode

df Frequency spacing of the power spectrum

$E_{i,w}$ Fraction of total energy associated with the i 'th mode in the w 'th window

e_n Fraction of total energy associated with the n 'th frequency in a mode

$f(n)$ The value of the circular frequency (Hz) at the n 'th frequency within a mode

H_n Amplitude of n 'th frequency component

I Total number of modes in the power spectrum

m_i The first statistical moment of the i 'th modal distribution

- N_i Number of frequency intervals of the i 'th mode
- $N_{i,w}$ Number of frequency intervals of the i 'th mode in the w 'th window
- P_i Probability of energy in the i 'th mode
- $S(n)$ Spectral offset magnitude at the n 'th frequency component
- s Weighted averaged sample standard deviation for the T -statistic
- s_i Combined sample standard deviation of the i 'th mode including both data windows
- $s_{i,w}$ Sample standard deviation of the i 'th mode in the w 'th window
- T_i T -statistic from the i 'th modal frequency difference between data windows
- $x(t)$ A time-history simulated from a target spectrum with random phase

ACKNOWLEDGMENTS

This work was supported by Texas A&M University and by the National Science Foundation, Division of Civil and Mechanical Systems under Agreement Number CMS-0428585. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the view of the National Science Foundation. Riser acceleration data was generously made available for this work by British Petroleum.

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