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Prediction and application of wave kinematics for near-shore structures subject to irregular seas with comparison to measured field data

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Abstract

A new hybrid method is proposed that combines stream function theory with spectral representation of an irregular wave process to predict design loads on an offshore wind turbine support structure. A particular difficulty in prediction of wave loading on near-shore structures for design purposes arises because the incident wave process is both highly non-linear and irregular. Generally, a designer must choose between prediction of wave kinematics by Stokes theory or stream function theory. Stokes theory can be used directly to predict irregular wave kinematics from a specified wave spectrum, but the predicted kinematics are not accurate in shallow water. Conversely, stream function theory accurately predicts wave kinematics for regular waves in shallow water, but it is not amenable for use with irregular waves. Here, both the irregularities and shallow-water effects are considered. Wave kinematics and loading predictions based on the new method are compared with those predicted by an existing method based on Stokes theory. Additionally, predicted wave loading based on the new method is compared with full-scale measured field data for an offshore wind-turbine support structure.

※Key words : Wave Kinematics, Stream Function Theory, Irregular Wave Loading, Wind Turbine

1. Introduction

Generation of electrical power by offshore wind turbines is experiencing a period of unprecedented growth, a pattern that seems likely to continue into the future. However, land near electricity markets suitable for turbine placement is scarce and future developments are hampered by concerns including turbine noise and appearance. Offshore locations alleviate these concerns and offer generally higher wind speeds, smaller wind gradients and less turbulence than onshore.

Unlike their onshore counterparts, offshore turbine support structures are subject to loading by ocean waves. Generally, offshore wind farms are located near-shore in intermediate water depths: depths typically between 5 and 15 meters. Processes for accurate prediction of

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wave kinematics in deep water have been developed in support of offshore oil and gas developments, while wave theories for kinematics in shallow water have been established in support of coastal engineering. Adequate design methodologies for intermediate water depths are still being developed. Prediction of wave kinematics in irregular seas provides a necessary step in prediction of wave loading.

The newly proposed method combines the strengths of the Stokes and stream function theories. Stokes theory is used to generate an irregular time-history of the free surface and then stream function theory is used to predict associated time-histories of wave kinematics. The resulting force time-history is next used to predict short- and long-term global load statistics useful for engineering design of these near-shore structures. Generation of the time-history, rather than a complete solution in the frequency domain, is necessary because of the highly non-linear nature of wave-loading.

2. Wave Theories

Wave kinematics must be determined to predict wave loading. A brief overview of Stokes theory for regular and irregular waves and of stream function theory is presented.

2.1. Regular Waves

Regular waves have a fixed, repeating period and a fixed amplitude. Linear wave theory is strictly valid for waves of infinitely small amplitude relative to wave length. Nonlinear effects, such as relatively higher crests and flatter troughs, become increasingly pronounced with increasing wave steepness and decreasing water depth.

2.1.1 Airy Theory

Airy wave theory, or first-order Stokes theory (e.g. Sarpkaya/Isaacson), is based on the Laplace equation:

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$$

in which ϕ is the velocity potential and coordinates x and z are in the direction of wave travel and vertically from the mean free-surface, respectively.

The boundary conditions are:

Bottom boundary condition (BBC),

$$\frac{\partial \phi}{\partial z} = 0 \quad at \quad z = -d$$

Kinematic and dynamic free surface boundary conditions (KFSBC, DFSBC),

$$\frac{\partial \phi}{\partial z} = \frac{\partial \eta}{\partial t} + \frac{\partial \phi}{\partial x}\frac{\partial \eta}{\partial x} \qquad at \quad z = \eta(x,t)$$

$$\frac{\partial \phi}{\partial t} + \frac{1}{2}\left\{\left(\frac{\partial \phi}{\partial x}\right)^2 + \left(\frac{\partial \phi}{\partial z}\right)^2\right\} + g\eta = const$$

$$at \quad z = \eta(x,t)$$

where d is water depth, η is the wave profile, and g is the gravitational acceleration. The additional conditions are periodic lateral boundary condition, and symmetry condition in the x and t directions.

Airy theory is commonly used for deep water, though it is not convergent for steep waves. Formally, Stokes expansion is valid for:

$$\frac{H}{d} \ll (kd)^2 \text{ for } kd < 1 \text{ and } \frac{H}{L} \ll 1$$

where H is wave height, d is water depth, L is wave length, and k is wave number (e.g. Chakrabarti). These limitations become increasingly restrictive in shallow water. Airy theory is also not directly applicable to prediction of water particle kinematics above the mean water level.

2.1.2 Stream Function Theory

Stream function theory provides an alternative approach which is not subject to some of the limitations of Airy theory (Dean, 1965).

The governing equation for stream function theory (e.g. Sarpkaya/Isaacson) is also the Laplace equation, but here the Laplace equation is solved along a streamline (ψ) :

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial z^2} = 0$$

With Bottom boundary condition,

$$\frac{\partial \psi}{\partial x} = 0 \quad at \quad z = -d$$

KFSBC,

$$\frac{\partial \psi}{\partial x} = -\frac{\partial \psi}{\partial z}\frac{\partial \eta}{\partial x} \quad at \quad z = \eta(x,t)$$

The other conditions are the same as those of Stokes theory. The solution for this problem is

$$\psi(x,z) = \sum_{n=1}^{N} X(n) \sinh\left(nk(d+z)\right) \cos nkx$$

where N is the order of the stream function theory and X(n) are the coefficients.

Dean conducted an extensive series of wave tank experiments which both validated stream function theory and established the order to which stream functions must be evaluated to accurately predict wave kinematics. Based on this work, Dean produced extensive tables of integral quantities and wave kinematics for 40 cases of wave height and period using stream functions up to the nineteenth order (Dean, 1974). These original integral quantities form the basis of the stream function kinematic prediction methodology applied here.

Wave kinematics predicted by stream function theory shows very good agreement with measured data. For regular waves, Chakrabarti concludes that measured wave-tank data correlate quite well with analytical results from stream function theory over a very broad range of water depths (Chakrabarti, 1980). Since Dean's original work, numerous attempts have been made to improve the solution to Dean's system of differential equations, but results have been found to be nearly identical to Dean's 1974 tabulations (Huang/Chen, 1993).

2.2.3 Comparison between Airy and Stream Function Theories

Airy and stream function theories are compared in Figure 1. The wave height is 6.1 m, the period is 5.5 seconds and the water depth is 13.6 m, which is very near the breaking limit. Note that the Airy wave is sinusoidal while the stream function wave has peaks which systematically exceed the troughs.

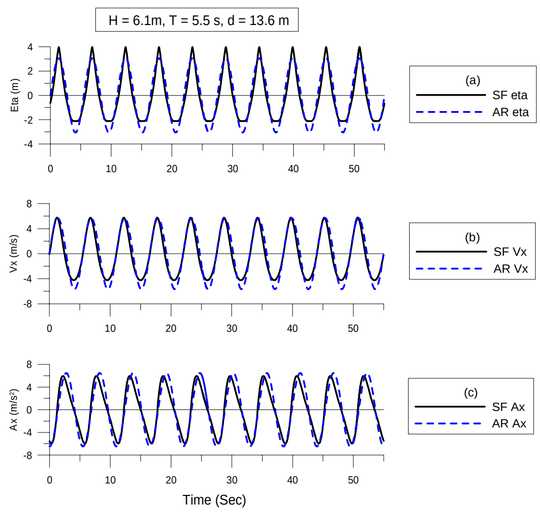

Fig. 1 Comparison Wave Profiles and Kinematics of Stream function and Airy theory at 8.76 m from bottom

2.3 Irregular Waves

Actual ocean waves generally have irregular profiles, and are of varying height and period.

2.3.1. Irregular Linear Waves

Irregular wave profiles are commonly represented as a sum of many linear regular waves with random phases, where the height of the wave component at each frequency, f_k , is determined from a specified energy density spectrum:

$$H_k = H(f_k) = 2\sqrt{2S(f_k)\Delta f} \quad \text{and} \quad T_k = 1/f_k$$

where Δf is the frequency spacing and $S(f_k)$ is the energy density. The phase angle, θ_k , is randomly chosen from a uniform distribution. The irregular surface elevation, $\eta_1(\mathrm{t})$, at a fixed spatial location is generated by superposition of all component waves.

$$\eta_1(t) = \sum_{k=1}^{N/2} A_k \cos(\omega_k t + \theta_k) = \sum_{k=1}^{N} Y_k \exp(i\omega_k t)$$

where the amplitudes of each component are

$$A_k = H(\omega_k)/2$$

and the angular frequency of each component is

$$\omega_k = 2\pi f_k = 2\pi / T_k$$

The exponential component and its complex conjugate are

$$Y_k = A_k \exp(i\theta_k)/2 \quad \text{for} \quad k \le N/2 , \quad \mathrm{Y_{N\text{-}k}}$$

Similarly, kinematics are obtained by summation of components. The total horizontal velocity *u* for a specified elevation is

$$u(x,z,t) = \sum_{i=1}^{n} \omega_i \frac{\cosh k_i(d+z)}{\sinh k_i d} \eta_i(x,t)$$

where *h* is water depth, and *n* is the number of spectral frequency components.

Applying this spectral representation to a first-order Stokes model has the obvious advantage of simplicity. Unfortunately, the resulting Gaussian process fails to preserve nonlinear effects, such as the marked asymmetry between wave crests and neighboring troughs. As in the regular-wave case, this asymmetry increases with decreasing water depth, which is problematic in the relatively shallow waters typical of offshore wind farms.

3. Wave-Force Prediction

Prediction of a structural design force requires both prediction of structural forces from wave kinematics and prediction of a design event, which can be either a design wave for which the forces are calculated or a direct prediction of a maximum force.

3.1 Wave Forces on Slender Structures

Wave forces on slender structures can be calculated as a combination of drag and inertial forces. Drag is caused by viscous effects which are related to water particle velocities and tend to dominate where the waves are long relative to diameter of the structure. Inertial forces are related to water particle accelerations and dominate when the structure is large compared with the wave length. The Morison equation (Morison et al, 1950) is commonly used to predict wave forces on slender structures. It predicts total force as a sum of drag and inertia:

$$F = 0.5\rho C_d A_p |U| U + \rho C_m \Delta \cdot dU/dt$$

where $A_p(= Ddz)$ is the projected frontal area; $\Delta(= 0.25\pi D^2 dz)$ is the displaced volume of the structure; and U is the horizontal water particle velocity. Wave kinematics vary with depth below the water surface. The total force on a vertical structure is predicted by integration of an array of Morison elements. These forces are integrated from the sea floor to dynamic free surface (stream function theory), or to the mean sea-level (Airy theory).

The Morison equation is applicable if the structure is sufficiently small compared to the wave length to leave the wave kinematics unaffected by the presence of the structure.

Generally, diffraction effects are negligible if $D < L/5$ (e.g. Sarpkaya/Isaacson). Turbine support structures are typically monopole structures with diameters of about 4m, for which application of the Morison equation is valid for most offshore design conditions.

3.2 Determination of Design Events

3.2.1 Design Wave Approach

In this very simple approach, a single extreme wave event is assumed to create the design wave-load on the structure. The design event is commonly often specified as having a 100-year return period. The design wave approach is desirable in many engineering applications because it is simple and requires only minimal calculations.

3.2.2 Frequency-Domain Approach

Frequency-domain approaches are not directly compatible with a Morison formulation because of the non-linearity introduced by the squaring of the water-particle velocity.

3.2.3 Time-Domain: Irregular Sea Simulation

In this approach, an irregular sea-state is simulated, and the associated wave-kinematics are applied to the Morison equation to predict a time-history of the wave-force. Statistical methods are then applied to the time-history to predict an extreme event. Use of the approach with Stokes theory is straightforward because of the ease with which the kinematics associated with the irregular sea-state can be predicted. Unfortunately, prediction of the kinematics associated with an irregular sea using stream-function theory is extremely difficult.

3.2.4 Wave-by-Wave Force Prediction

In this approach, individual waves are isolated within the time-history of irregular wave profiles. This time-history could in concept be measured time-history or could be simulated using Stokes theory. Wherever the time-history is obtained, each wave period, *T,* is defined as the duration between free-surface up-crossings of the mean. The wave height (*H*) is the difference between the maximum and minimum elevations reached during this period. The kinematics associated with that wave are assumed to be the same as with a regular wave with the same wave height, period, and water depth. The purpose of applying the wave-by-wave method is that these kinematics can then be accurately predicted using stream function theory. Figure 2 shows the simplification of the wave profile implicit to the wave-by-wave method. Note that wave-length appears as time, because the wave is measured by a probe at a fixed location.

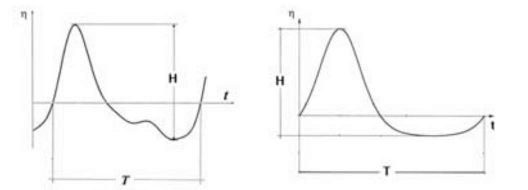

Fig. 2 Irregular and simplified wave profiles

3.2.5 Summary of the New Hybrid Method

In the new hybrid method, a time-history of a long-crested sea surface is first simulated from a specified wave spectrum. Here, an actual measured sea-state was used rather than a simulation so the effectiveness of the new method could be assessed by direct comparison with measured data; generally, designers do not have access to a measured wave profile representing the design condition. Whether the sea surface profile is measured or simulated, it is next segmented into a series of individual waves, which are defined as being between up-crossings of the mean free surface at a fixed spatial location. The kinematics of each individual wave is then predicted at an array of elevations above the sea floor using stream function theory over the entire wavelength. The velocities and accelerations are then re-assembled to produce a series of time histories of water particle velocities and accelerations at a series of fixed elevations along the structure. The Morison equation is next applied to produce an array of predicted forces along the structure that are integrated to predict a time-history of bending moments at the base. Finally, the statistical moments of the bending-moment time-history are calculated and applied to the Hermite transform (e.g. Winterstein) to predict the expected maximum bending moment.

4. Comparison with Measured Data

Measured Full-scale data are available for comparison with predicted results. The data were collected as part of the OWTES project (OWTES) between November 2001 and November 2002 at an offshore wind turbine at Blyth, North-East of England. It is situated approximately 1 km offshore in a mean water depth of about 9 m. Full scale measurements have been collected at 20 Hz in a series of 30 minute campaign datasets. Two datasets are used in this comparison. The first has significant wave-height of 6.1 m and a tidal level 4.6 m above the mean water depth, and the second has a 2.8 meter significant wave height and a tidal level 4.5 m above the mean water depth.

4.1. Comparison of Wave Profiles

Table 2 presents a statistical comparison of the two campaigns. The third and fourth statistical moments indicate the non-Gaussianity of the wave process: a Gaussian process has skewness equal to zero and kurtosis equal to three. The skewness is indicative of asymmetry in the wave profile, with systematically larger peaks and shallower troughs. As expected, the 6.1 m sea-state has significantly higher skewness than the 2.8 m sea-state. The minimum and maximum are those observed in each 30-minute dataset.

Table 2 Comparison of Wave Profiles

	Hs = 6.1 m	Hs = 2.8 m
Std Dev	1.5192	0.6911
Skewness	0.8997	0.2676
Kurtosis	3.3494	3.3834
Min (obs)	-0.2560	-0.1672
Max (obs)	9.7661	6.9586

The wave-by-wave method uses only individual wave heights and periods to predict wave kinematics, so local variations in the wave profiles are neglected in the wave-force calculations. The profile assumed for the wave-by-wave method is compared with measured profile in Figure 3.

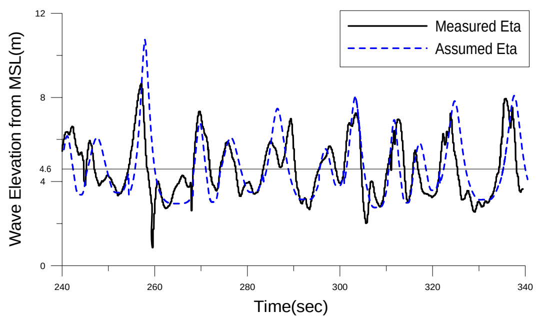

Fig. 3 Comparison between measured wave profile and that used for wave-by-wave calculations (Hs= 6.1m)

4.2. Comparison of Wave Loading

Data measured at the Blyth wind farm include a time-history of total bending moments at the base of the structure and at the mean sea level. These measured moments include other effects: wind forces on the upper part of the structure and blade rotation. Bending moments due to only wave loading are estimated by subtracting the bending moments at mean sea level from those at the mud-line. Unfortunately, this process also removes hydrodynamic loads due to waves above the mean sea level, which are believed to be significant.

Figure 4 shows a time-history of measure and predicted bending moments at the base of the structure. The measured and predicted bending moments follow the same general pattern, though individual maxima and minima are sometimes either under- or over-predicted.

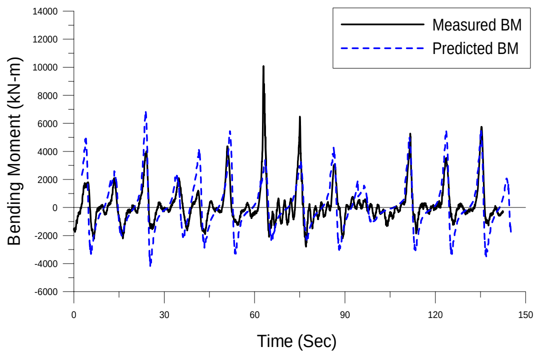

Fig. 4 Comparison of measured and predicted bending moment at mudline

These bending moment predictions are based on a 4 meter diameter monopole and Morison coefficients C_d and C_m of 0.55 and 1.33, respectively (Chakrabarti, 1987). Current forces are neglected in the predictions. Wave forces on the structure are inertia dominated, so current-induced drag is expected to contribute very little to total bending-moment. Figure 5 shows the extent to which inertial forces dominate the wave loading by comparing the inertial term in the Morison equation with the total of the drag and inertial terms.

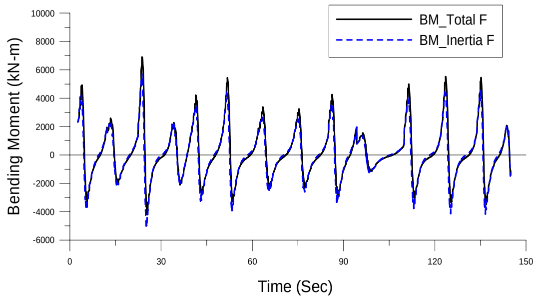

Fig. 5 Comparison of total bending moments and bending moments due to only inertia forces in campaign Hs = 6.1 m

4.3. Predicted versus Measured Moments

4.3.1 Results from Design Wave Analysis

The maximum wave height observed in the 30 minute realization was 9.04 m with an up-crossing period of 5.8 seconds, which is well above the stream function breaking limit of 7.7 m. Neither Airy nor stream function theory can be meaningfully used to predict kinematics for waves above the breaking limit. The wave is instead assumed to be at the breaking limit, which fixes the period at 6.8 seconds. Wave kinematics associated with this single wave are calculated using both stream function and Airy theories, and forces are calculated using the Morison formulation. The maximum bending moment predicted using stream-functions is 11585 kN-m; the maximum predicted using Airy theory is 6294 kN-m. The maximum bending moment predicted by stream functions compares very well with the measured maximum of 11793 kN-m.

4.3.2 Results from Irregular sea-state Analysis

The load for which a structure will be designed is generally based on a statistical analysis to predict the maximum load the structure will experience in its design life. A prediction of the largest load the structure would be expected to experience in 100 years of service is commonly used. For design purposes, accurately predicting overall statistics of the loading process is far more important than accurate predictions of the loading by individual waves.

Table 3 provides a statistical comparison of results from the predicted bending-moment time-history with measured data. The standard deviation represents the energy of the process, and the standard deviation of the predicted process exceeds the observed by about 22%. This higher prediction is expected because the prediction accounts for wave forces above the mean water level, while the measured data do not. Standard deviations predicted by Airy theory and by the new hybrid method are nearly in agreement.

Table 3 Statistical Analysis of Bending Moments

Hs = 6.1 m	Measured	Predicted Hybrid Method	Predicted Airy
Mean	-7.0	115.0	1.87
Std. Dev.	1314	1597.	1518.
Skewness	2.2870	0.6812	0.0002
Kurtosis	11.799	4.774	2.557
Max (obs)	11793.	10799.	6051.
Max (pred)	11652.	10740.	5416.

The skewness measures asymmetry in the process, with positive skewness indicating systematically higher peaks and shallower troughs. The hybrid method predicts positive skewness, but not nearly as high as that of the measured data. Similarly, the measured coefficient of kurtosis is nearly 2.5 times the predicted value. The authors speculate that the relative difference between the predicted and observed skewness and kurtosis may be caused by breaking waves impacting the structure. Breaking waves create very large loads and the associated kinematics are not well predicted by stream-function theory.

Wave-loading for design purposes can be predicted from the statistical moments of the bending moments using the Hermite transformation (e.g. Winterstein). Here, the Hermite transformation is used to predict the maximum wave load which should be expected in a 30 minute realization of the sea state; this value is directly comparable to the ``observed" maximum, but it provides a more robust estimator of the expected maximum than a single realization. The ``Max (obs)" bending moment shown in the table is the largest of all values calculated in the wave-by-wave method for one realization of the sea-state; in this case, the realization was 30 minutes of measured wave data. The second maximum shown (``Max (pred)") is the maximum predicted using the Hermite transformation (Winterstein 1988, Winterstein et al 1994).

Little difference is observed between the observed and statistically predicted maxima. The extreme load predicted by the new method under-predicts the equivalent maximum from the observed data by about 9%. The over-prediction of standard deviation coupled with the under-prediction of the maxima shows the importance of the non-Gaussian nature of the process. The large skewness and kurtosis lead to large maxima; the hybrid method under-predicts the skewness and dramatically under-predicts the kurtosis, which leads to an under-prediction of the maxima, despite over-predicting the standard deviation. The prediction based on the more traditional Airy theory under-predicts the maximum by over 50% despite the predicted standard deviations' being nearly identical.

5. Conclusions and Recommendations

A new hybrid method for predicting design loading on near-shore structures has been presented. The new method has been compared with measured field data and found to provide far better predictions of the maximum loading than a more traditional method based on Airy theory (Table 3). The more traditional method is observed to under-predict actual forces by over 50%.

In the new method, a time-history of the sea surface is first simulated from a specified wave spectrum. The kinematics of individual waves are then predicted using stream function theory. The resulting velocities and accelerations are used in a Morison formulation to predict a time-history of bending moments at the base. Finally, the Hermite statistical model is applied to predict design wave-loading on the structure. Here, actual measured wave time-histories were substituted for the simulated data enabling meaningful comparison between predictions and measurements.

The difference between predicted and observed maximum bending moments is small (9%), but the authors believe both values under-predict actual wave loading. The measured data under-predicts because the data does not include wave loading above the mean sea level. The new hybrid method under-predicts because it underestimates the non-Gaussian nature of the process (kurtosis value of 4.8 predicted vs 11.8 measured). The associated under-prediction of the extreme design values is at least in part because waves in the measured or simulated time histories found to exceed the breaking limit are reduced in height to be at the breaking limit so stream function theory can be applied. Extreme waves exceeding the breaking limit were found in the measured time-history. Future work may directly consider the effects of breaking waves.

Surprisingly, the traditional design-wave method was found to yield the most accurate prediction of design loading. This method predicted a bending moment of 11585 kN-m, compared with a measured moment of 11793, which is a difference of only 2%. The reader is again cautioned that the authors believe both the predicted and measured bending moments under-predict the actual wave loading on the structure.

The new methodology was found to yield substantially better force predictions than a wave-by-wave approach using Airy theory, but it was not found to yield better predictions than the comparatively simple design-wave method. However, the authors believe this work lays a firm foundation for future improvements. In particular, directly addressing loading due to breaking waves is expected to significantly improve prediction accuracy.

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