

# The 2nd Texas A&M at Galveston Mathematics Olympiad

September 24, 2010

Name:

UIN:

Email:

**Instruction: Show steps or reason to support your answer. Calculator is allowed. You may get the cash reward by solving some of the following questions.**

1. A runner passes 5 poles in 30 seconds. How long will he take to pass 10 poles? The consecutive poles are of equal distance from each other.
2. A 12 hour clock shows 1 o'clock now. Exactly when will the hour and minute hands next coincide?
3. There were originally 7 pieces of papers in a pile. Somebody picked up a piece and cut it into 7 pieces and put them back into the pile again. This was repeated many times. Then, somebody counted the pieces of the papers in the pile and got the number 2010. Prove that he made a mistake counting.
4. What is the last digit of  $2^{2010}$ ? And what is the first digit of  $2^{2010}$ ?
5. Prove that  $\frac{n(n+1)(2n+1)}{6}$  is an integer, for any integer n.
6. Let  $f(x) = \frac{x}{\sqrt{1+x^2}}$ ,  $f_n(x) = f(f(\dots f(x)))$  is the nth composition of  $f(x)$ . Find  $f_{99}(1)$ .

(Composition or superposition of two functions  $f(g(x))$  means: plug  $g(x)$  as an argument into  $f(x)$ .)

7. Numbers  $p$  and  $8p^2 + 1$  are both primes. Prove  $8p^2 - p + 2$  is prime.

(Prime number is an integer  $> 1$  which is divisible only by 1 and itself.)

8. Write down the first 3 significant digits of  $\sqrt{0.999\dots 9}$  with a hundred 9's. Show your work.

(Significant digits: the digits which start from the first non-zero digit after the decimal point.)

9. Given any 50 integers, prove that you can choose some of them so that their sum is divisible by 50.

### **Helpful Notes:**

#### **<The Dirichlet Principle>**

Suppose we have at least  $n+1$  people living in  $n$  houses. The Dirichlet principle says that in this case, there should be a house with at least 2 people in it.

#### **<The Remainder Theorem>**

For any positive integers  $a \geq b$ , we can find unique integers  $k$  and  $r$  such that  $a = kb + r$ , where  $0 \leq r < b$ . eg, when dividing 205 by 3, we will have 68 as the quotient and 1 as the remainder, that means

$$205 = 68 \times 3 + 1.$$