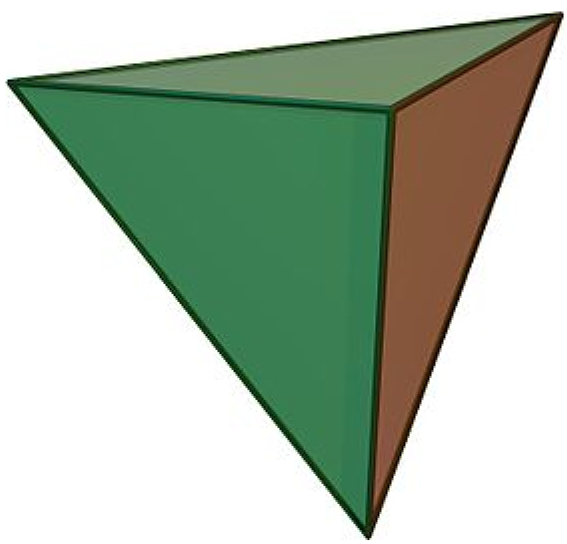
The 9th Texas A&M at Galveston Mathematics Olympiad.

You need to offer a comprehensive and detailed proof to all your solutions.

Use separate sheets to write your solutions on. Staple all your work and print your name clearly.

1. Arithmetic progression is a sequence of numbers such as the next one is greater than the previous one by a fixed amount, e.g. 3,7,11,15,19,…
   Let $a, a+d, a+2d, a+3d, \ldots$ be an infinite arithmetic progression of integers. Prove that the sums of digits of these numbers can never form an arithmetic progression, even if we rearrange the numbers.
2. A regular tetrahedron is a polyhedron with 4 vertices whose faces are equilateral triangles.
   Given a regular tetrahedron, we form a second tetrahedron inscribed into the first one which has as its vertices the centers of the faces of the first tetrahedron. We continue this process until we form the thousandth tetrahedron. If the volume of the first tetrahedron is 1, what is the volume of the thousandth tetrahedron?



3. In a circle of diameter X there are two perpendicular chords AB and CD. Prove that $AC^2 + BD^2 = X^2$
4. Given a sequence of Fibbonacci numbers,
   $F_0 = 1, F_1 = 1, F_2 = 2, F_3 = 3, F_4 = 5, F_5 = 8$, etc
   Prove that F_n >1000000 if $\frac{6 \ln 10}{\ln(3/2)} < n$.

5. Solve the following equation: $4^x + 16^x = 64^x$
6. JoAnn ate 100 candies in 76 consecutive days, and in every one of these 76 days she ate at least one candy. Prove that there were three consecutive days during which she ate just one candy a day.
7. Given 38 integers, none of them divisible by 75, prove that there are two of them with either sum or difference divisible by 75.
8. At a party everyone has met exactly 5 people. Show that the number of people at the party was even.
9. Prove that $11^{10} - 1$ is divisible by 100 without computing this number.
10. How many divisors does the number $2^2 3^3 5^5 4^4$ have?