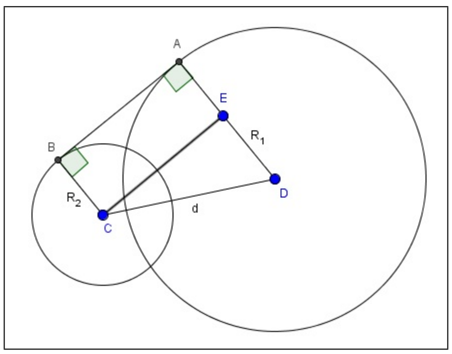
1. Given two intersecting circles of radii R_1 and R_2 such that the distance between their centers is d and a straight line tangent to both circles at points A and B. Find the length of the segment AB.

   Solution: (See figure below for references) Tangent lines to circles are perpendicular to their radii. Thus AB with the radii and the segment contacting the two circles' centers creates a quadrilateral with two right angles at A and B. A line through C (The center of one of the circles if they have equal radii or the center of the smaller circle) parallel to AB and hitting the other radius at E creates a rectangle $ABCE$ and a right triangle CED. Note that CE and AB have the same length. Thus from the Pythagorean Theorem we have:

   $$AB = \sqrt{d^2 - (R_1 - R_2)^2}$$

2. What are the last two digits of 2^{2012}?

   Solution: Note that there are only 100 possibilities for the last two digits, 00 to 99, so it must start repeating. We will list the last two digits of the powers of two, starting with 1, to determine the pattern that emerges. The pattern should be apparent by 2^{102}. Also, notice that multiplying the previous power of two's last two digits by 2 will provide the last two digits of the new power of two. Thus we do not need to find the actual power of 2.

$2^1 \to 2$	$2^2 \to 4$	$2^3 \to 8$	$2^4 \to 16$	$2^5 \to 32$	$2^6 \to 64$
$2^7 \to 28$	$2^8 \to 56$	$2^9 \to 12$	$2^{10} \to 24$	$2^{11} \to 48$	$2^{12} \to 96$
$2^{13} \to 92$	$2^{14} \to 84$	$2^{15} \to 68$	$2^{16} \to 36$	$2^{17} \to 72$	$2^{18} \to 44$
$2^{19} \to 88$	$2^{20} \to 76$	$2^{21} \to 52$	$2^{22} \to 04$		

   We see that the last two digits start over at 2^{22}. We can also see that 02 will never happen again after 2^1 as the pattern will provide 52 instead. Thus the pattern really starts over at 2^{21} and repeats every 20. Thus we divide 2012 by 20 and look at the remainder, 12, which means that we should look at the twelfth power of 2 for the last two digits. Thus the last two digits of 2^{2012} is 96.

3. Can a plane intersecting a cube result in a regular pentagon? Give the reasoning.

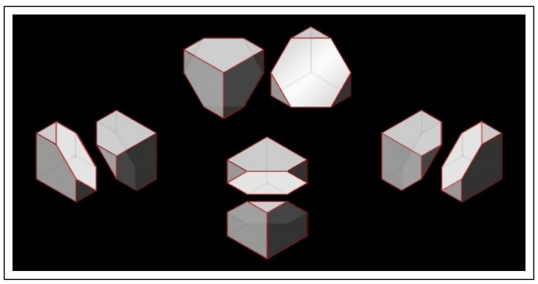
   Solution: The answer is no. The sides of the pentagon will be made from the faces of the cube. The cube has three pairs of opposite faces that are parallel. Since a cube has six faces any pentagon created from cutting the cube would have at least two pairs of parallel sides. However a regular pentagon has no sides that are parallel.

4. Given the Fibonacci sequence 1,1,2,3,5,8,13,21,...etc. show that there are more than 1,000 Fibonacci numbers smaller than 2^{1000}.

   Solution: We can pair each power of 2 with a Fibonacci number in an orderly fashion (i.e. 2^1 paired with 1, 2^2 paired with 1, 2^3 paired with 2, and so on). Notice that 2^{n+1} doubles the value of 2^n but that the Fibonacci numbers add the previous values to get a new value. Thus, after the third Fibonacci number, the $n+1$ Fibonacci number is less than doubling the n one. Since 8 is the 6th Fibonacci number and is less than $2^4 = 16$, there will be at least 1002 number less than 2^{1000}. Thus there are more than 1000 Fibonacci numbers less than 2^{1000}.

5. Let Q be a cube and let D be its diagonal connecting two vertices not belonging to the same face of the cube. What figure do we get if we cut the surface of the cube with the plane perpendicular to D and passing through the center of the cube?

   Solution: Regular hexagon.

6. Given 3 types of pumps: when a type-I pump pumps water into the empty pool it takes 3 hours to fill it, type-II pump takes 4 hours to fill it and type-III pump takes 5 hours. Also, the pumps pump the water out at the same speed as they pump it in. Suppose one has several pumps of each kind, some of the pumps of any type are set to pump water in and some of them are set to pump water out of the pool. Show that whatever the setup is, it is not possible to get them to fill the empty pool with water in exactly 7 hours if all pumps start and finish at the same time.

   Solution: *Coming Soon*

7. Suppose there are n boxes in which k balls are placed, where k and n are positive integers greater than 2. Given some positive integer r,

   (a) find the largest value of k which guarantees, that no matter how you place these k balls in n boxes, there would be at least one box containing no more than r balls.

   (b) find the largest value of k which guarantees, that no matter how you place these k balls in n boxes, there would be at least three boxes each containing no more than r balls.

   Solution:

   (a) The answer is $k = nr + (n-1)$. To see this consider that to get the most amount of balls we first need to fill every box with r balls. That gives us nr balls. Currently no box contains more than r balls. Next we put one more ball in every box except one of them. That is $n-1$ more balls. Note that we still have one box with only r balls. If we try to take a ball from another box and put it in this one then the box we took the ball from will have r balls. Also, if we add one more ball then we can put that ball in the box with r balls and thus every box has more than r balls.

(b) The answer is $k = (n-2)r + (n-3)$. (Note $n \geq 3$) This time, instead of filling every box with r balls, we will leave two boxes empty. This gives us $(n-2)r$ balls. Then using the same reasoning we put one more ball in every box with a ball except one of them. Thus we have three boxes with no more than r balls. Note that if we add one more ball we can put that ball into the box with r balls and thus we would have $n-2$ boxes with more than r balls and only 2 boxes with r or less balls.

8. A train shuttles between city A and city B at a uniform speed, spending negligible time at the stations and negligible time for acceleration and deceleration. A person living by the railroad tracks looks out the window at random times and waits until he sees the first train. He observes that on average, he sees the train coming from city B twice as often as from city A. What is the ratio of his distance from city B to his distance from city A?

   Solution: *Coming Soon*

9. Given a group of 101 telephones such that each telephone is connected with exactly two others in this group. Show that there is an odd number k such that $k > 2$ and a set of k telephones $T_1, T_2, ..., T_k$ from this group such that this set is connected in a circular fashion, i.e. first telephone T_1 is connected to T_2, T_2 is connected to T_3, etc., T_{k-1} is connected to T_k, and T_k is connected to T_1.

   Solution: *Coming Soon*

10. Given a rectangular tabular arrangement of numbers (a matrix) with n rows and m columns. Each number in this table is an integer, negative, positive, or zero. One is allowed to perform the following operations:

    (a) Simultaneously change the signs of all numbers in any particular column

    (b) Simultaneously change the signs of all numbers in any particular row

    Prove that after several such operations, we can get the sum of numbers in each row to be nonnegative and the sum of numbers in each column to be nonnegative at the same time.

    Solution: *Coming Soon*