The Fifth Mathematics Olympiad at Texas A&M at Galveston
October 23, 2013

1. Given a straight line L on a plane and two points A and B which are not lying on the line L, (they may be on the same side of L or different sides of L). Find a point C on the line L such that the sum of distances CA+CB is minimal.

2. Solve the equation in positive integers, where x is a prime: $2x + 1 = y^3$.

3. Let there be a rectangle having a rectangular hole inside and let us denote the resulting figure by X, (the hole and the original rectangle do not have to be concentric.) Draw a straight line which divides the figure X into two parts of equal area. Describe how the straight line is drawn.

4. A motor boat went from city A to city B and back on the lake with still water going a constant speed. After that, it went the same distance and back on the river which had a current with a constant velocity in one direction. During the second trip in the river, the boat had the same constant speed as in the first trip in the still water. During the second trip, the boat took 10% longer to complete the trip. What is the ratio of the velocity of the current and the speed of the boat.

5. Prove that out of 15 integers one can choose 2 whose difference is divisible by 14.

6. Simplify the expression $\sqrt{30\left(4 - \sqrt{15}\right)}$

7. Given a deck of 52 cards, what is the minimal number of cards that needs to be drawn to guarantee that there will be at least 7 cards of the same suit?

8. Eight line segments, with lengths no greater than 20 inches, and no shorter than 1 inch, are given. Show that one can choose three of them to represent the sides of a triangle.

9. A certain airline has one airplane in each of the several supporting airports located at different distances from one another. Each of these airplanes takes off and lands at the nearest supporting airport. Prove there will be no more than 5 airplanes in any of the supporting airports when all the airplanes land.