

The Third Mathematics Olympiad at Texas A&M at Galveston
October 5, 2011

1. Between noon and the succeeding midnight, how many times do the hands of a clock form a angle? Give the reason.

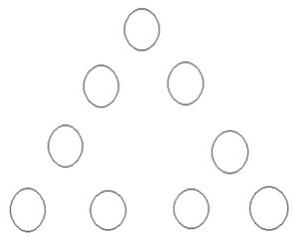
2. There is a solid rectangular box with dimensions $300 \times 200 \times 100$. Someone painted its entire surface red; then cut it into $1 \times 1 \times 1$ cubes. Find the numbers of cubes: (a) free of paint; (b) painted on exactly one side; (c) painted on exactly two sides; and (d) painted on exactly three sides.

3. Find the digits such that when they are substituted for the letters below, you get a correct identity:

$$\begin{array}{r}
 F \ O \ R \ T \ Y \\
 + \quad \quad T \ E \ N \\
 + \quad \quad T \ E \ N \\
 \hline
 S \ I \ X \ T \ Y
 \end{array}$$

4. Show that if n and p are integers, then the product $n(n + 1) \dots (n + p)$ is divisible by $p + 1$.

5. Put the numbers 1,2,3,4,5,6,7,8,9 into the following circles and make the sum of the numbers on each side of the triangle equal. You can use each number only once to fill all the circles.



6. A train passes by an observer in t seconds and by a bridge of length L in s seconds. Find the speed of the train and its length in terms of t , s , and L . (Assume that the train passes through the bridge from the moment when the beginning of the locomotive enters the bridge until the moment when the end of the last car leaves the bridge)

7. Show that no plane can intersect more than 4 edges of a tetrahedron.

8. Find all triples of integers x , y , & z such that $x^3 + 3y^3 = 9z^3$. (Consider all possibilities for the integers, positive, negative, or 0.)

9. Consider a sequence of numbers a_1, a_2, \dots, a_n each of which is either $+1$ or -1 , and let us transform them into another sequence of $+1$ s and -1 s by multiplying a_1 by a_2 , a_2 by a_3 , etc. finally multiplying a_n by a_1 .
- (a) Show that if $n = 2^k$ then after several transformations we should get all $+1$ s.
 - (b) Show that if n is odd then we can never get all $+1$ s unless the original set consists of all $+1$ s, or all -1 s